CRINES

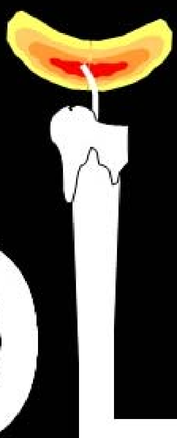
Light a CANDLE

An Innovative Burnup Strategy of Nuclear Reactors

Second edition

Hiroshi Sekimoto

與其詛咒黑暗
不如點亮蠟燭
中國諺語



CANDLE

Better to light a candle than curse the darkness

http://www.crines.titech.ac.jp/eng/

Light a CANDLE

An Innovative Burnup Strategy of Nuclear Reactors

Second edition

Hiroshi Sekimoto

Center for Research into Innovative Nuclear Energy Systems
Tokyo Institute of Technology
2-12-1-N1-35, Ookayama, Meguro-ku, Tokyo, 152-8550
JAPAN
E-mail: hsekimot@ crines.titech.ac.jp

Center for Research into Innovative Nuclear Energy Systems (CRINES)
Tokyo Institute of Technology
URL: http://www.crines.titech.ac.jp/eng/

ISBN978-4-905205-00-5 C3053 E

Preface to Second Edition

The first edition of "Light a CANDLE" was published in 2005 with the support of COE-INES (this program is described briefly in the preface to the first edition). New research results on CANDLE reactors have subsequently been obtained. Bill Gates recently supported TerraPower's Travelling Wave Reactor, which employs a breed-and-burn strategy similar to CANDLE burning. Consequently, many more people have become interested in CANDLE burning. Personally, I intend to retire soon from Tokyo Institute of Technology. These are the main reasons for the revision of this book.

Unlike conventional nuclear reactors, the CANDLE reactor does not require excess reactivity to continue burning. In addition, its power distribution and core characteristics do not vary as burning progresses. Furthermore, if this burning strategy is applied to a large fast reactor that has an excellent neutron economy, it will be possible to use depleted or natural uranium as an exchange fuel so that 40% of the fuel can be utilized. This burning strategy has many potential advantages in terms of safety, waste reduction, safeguarding and nonproliferation of nuclear materials, and fuel sustainability.

CANDLE burning can be used to realize a small, long-life reactor that satisfies the requirements of safety, waste reduction, safeguarding and nonproliferation of nuclear materials, and fuel sustainability. If we apply CANDLE burning to a large fast reactor with metallic fuel and sodium coolant, we can design an excellent nuclear reactor that satisfies economic requirements as well as the requirements of safety, waste reduction, safeguarding and nonproliferation of nuclear materials, and fuel sustainability.

Several problems need to be overcome before such high-performance reactors can become a reality. However, these problems appear to be relatively easy to solve and they should be overcome in the near future. I hope that many young scientists and engineers will read this book and that some of them will attempt to solve these problems so that many high-performance nuclear reactors will be constructed.

I should add the following names of my students to the list shown in the Preface to First Edition as the contributors to this book: Mr. Seiichi Miyashita, Mr. Mitsuyoshi Kasahara, Dr. Akito Nagata, Dr. Ming-Yu Yan, Dr. Tsuyoshi Ohkawa, Mr. Hiroshi Taguchi, and Mr. Sinsuke Nakayama. I am very grateful to them, and also grateful to Ms. Mariko Hiraishi for her help as my secretary.

Tokyo Hiroshi Sekimoto

October 2010

Preface to First Edition

This world is created in an orderly fashion. With the advancement of science, it is becoming increasingly clear what the purpose behind this order is. It almost appears as if the world is created in an orderly fashion for the benefit of humankind. Nuclear fission provides a good example of this. The neutrons generated in the process of nuclear fission can be used to trigger succeeding nuclear fissions or to create further fissile material. Very few neutrons are left over in this process. How should we use these remaining neutrons? The Creator of this world has presented us with this very interesting question and seems to be wondering what solutions we come up with. CANDLE burnup is one solution.

CANDLE is a new burnup strategy for nuclear reactors. The acronym stands for <u>C</u>onstant <u>A</u>xial Shape of <u>N</u>eutron Flux, Nuclide Densities and Power Shape <u>D</u>uring <u>L</u>ife of <u>E</u>nergy Production, but also represents the candle-like burnup. When this burnup strategy is adopted, although the fuel is fixed in a reactor core, the burning region moves, at a speed proportionate to the power output, along the direction of the core axis without changing the spatial distribution of the number density of the nuclides, neutron flux, and power density. The reactivity and reactor characteristics do not change. Most significantly, when using this strategy it is not necessary to use control rods for the control of the burnup. A CANDLE nuclear reactor is hence safer, and just as importantly, makes us feel safer.

CANDLE burnup has various other ground-breaking merits. When this burnup is used in a fast reactor that has excellent neutron economy, excellent performance is obtained. It is possible to use natural uranium or depleted uranium as fuel and about 40% of the fuel will burn. A large amount of depleted uranium is already available, and hence if we are able to use it as fuel, we can continue to use nuclear energy for almost a millennium without further uranium mining, enrichment, and reprocessing. In addition, the amount of spent fuel is greatly reduced.

While there are great advantages in using CANDLE burnup, numerous technological developments are necessary before it can be used. However, for block-fuel high-temperature gas-cooled reactors, currently under development in several countries, CANDLE burnup can be applied without additional technological development. In this booklet, the specific application of CANDLE burnup to a high-temperature gas-cooled reactor and a fast reactor with excellent neutron economy are described.

When the former Nuclear Regulatory Commission Chairman Dr. Meserve lectured on the current status of nuclear energy, he cited a Chinese proverb to brighten the present dark status. I remember he said, “Better to light a candle than curse the darkness”. Thus, I have given the booklet the title: “Light a CANDLE”. I hope that this booklet will contribute to the bright future of nuclear energy.

I have avoided rigorous discussions in this booklet so that it can be read in a relaxed manner. If this makes it difficult for experts to understand, then please forgive me. I recommend that interested experts should read the references. Even though numerous papers concerning CANDLE burnup have been published, they are not targeted to the general audience, and therefore I have not listed many references. Although I tried not to use equations, I had no option in the explanation of the analysis method, and differential equations had to be included, though I used only the most basic equations from nuclear reactor theory. Those who have studied the subject will easily understand these equations, however, those readers who are not good at mathematics can skip that chapter. This booklet is written so that even those readers can understand the rest of the chapters.

I have received encouragement from numerous people in preparing this booklet. Professor Thomas H. Pigford, my Ph.D. thesis adviser, is chief amongst them. He has an interest in the important role of the combination of neutron transport and burnup, which was the topic of my Ph.D. thesis, and gave me great encouragement in my research. Professor Ehud Greenspan assisted with considerable discussions concerning CANDLE burnup. It was he who informed me of similar research conducted by Dr. Edward Teller. I am also grateful to the numerous other researchers for giving me advice and encouragement.

Although I do not know Dr. Alvin M. Weinberg personally, I once sent him a

paper, as he had been promoting the development of inherently safe reactors and I thought that he would be interested in CANDLE burnup. Dr. Weinberg showed an interest in the paper and sent me a letter of encouragement. I heard that he contacted Dr. Teller. Some time later I saw Dr. Teller's obituary in the newspaper. I would have liked to have known what he thought of CANDLE burnup.

The Japanese Minister of Education, Culture, Sports, Science and Technology began "21st Century COE (Center of Excellence) Program" in fiscal 2002 for selecting excellent research institutes of universities and forming internationally competitive research bases. Academic disciplines from humanities and social sciences to natural sciences are divided into ten categories. A proposal from Tokyo Institute of Technology "Innovative Nuclear Energy Systems for Sustainable Development of the World (COE-INES)" was adopted in the category of "Mechanical, civil, architectural and other fields of engineering." It is the only one COE in the nuclear engineering field. CANDLE burnup is one of the most important research topics in COE-INES.

The research described in this booklet was conducted by Dr. Kouichi Ryu, Mr. Kentaro Tanaka, Mr. Takashi Takada, Dr. Yasunori Ohoka, Mr. Yutaka Udagawa, Mr. Ken Tomita, and Mr. Makoto Yamasaki, graduate students of my research laboratory. I am very grateful to them, and also grateful to Associate Professor Tohru Obara for his fruitful discussions.

Tokyo Hiroshi Sekimoto

November 2005

Contents

Preface to Second Edition i
Preface to First Edition iii

1. Excess Neutrons 1
   1.1. Does Instability Imply Greater Stability? 1
   1.2. Nuclear Fission 2
   1.3. Chain Reactions and Criticality Control 4
   1.4. Burnup and Burning Control 8
   1.5. Neutron Economy, Use of Excess Neutrons 9

2. What is the CANDLE Burnup Strategy? 12
   2.1. Concept of the CANDLE Burnup Strategy 12
      2.1.1. Hard-Spectrum Fast Reactors 12
      2.1.2. Thermal Reactors with Burnable Poison 17
   2.2. Advantages of and Problems with the Burnup Strategy 20

3. Mathematical Explanation and Analysis Method 26
   3.1. Basic Equations of Neutron Transport and Fuel Burning 26
   3.2. Steady State CANDLE Burnup Equation 28
   3.3. Approximated Treatment 31
   3.4. Numerical Analysis of Steady State CANDLE burning 35
   3.5. Calculation System Employed in This Book 38

4. Parametric Studies of CANDLE Reactors in Equilibrium State 41
   4.1. Reference Design 41
   4.2. Parametric Surveys 42

4.3. Advantages and Problems ····· 44

5. Future Ideal Nuclear Energy System ····· 48
5.1. Requirements for Nuclear Reactors ····· 48
5.2. How CANDLE Reactors Satisfy these Requirements ····· 49

6. Small Long-Life CANDLE Reactors ····· 54
6.1. Small Long-Life Reactors ····· 54
6.2. Coolant Characteristics ····· 56
6.2.1. Lead and Lead-Bismuth ····· 56
6.2.2. ^{208}Pb Coolant ····· 58
6.3. Small Long-Life CANDLE Reactors Using Metallic Fuel and ^{208}Pb Coolant ····· 59

7. Initiation of CANDLE Burning ····· 61
7.1. Several Methods for Attaining Equilibrium State of CANDLE Burning ····· 61
7.2. Initial Core Composed of Enriched Uranium ····· 62

8. Problems with High Burnup ····· 65
8.1. Recladding ····· 65
8.2. MOTTO Cycle ····· 67

9. Increasing the Power Density ····· 71
9.1. Power Flattening Using Thorium ····· 71
9.2. Demonstrations ····· 72

10. History of Studies on CANDLE and Similar Burnups ····· 75
10.1. History of Studies on CANDLE and Similar Burnups up to 2008 ····· 75
10.2. Personal History ····· 76

Appendix

A1. Block-fuel HTGR ········ 85
A1.1. Principle ········ 85
A1.2. Advantages ········ 86
A1.3. Analysis Results ········ 87

A2. Technical Terms ········ 89

References ········ 95

1. Excess Neutrons

1.1. Does Instability Imply Greater Stability?

When I was a student, I assumed that a neutral state would be more stable than a state in which positive and negative charges were separated. I was thus very surprised to learn that the neutron is unstable but the proton is stable. However, I was impressed with the mechanisms of nature when I realized that the instability of the neutron is essential for our existence in the universe. An isolated neutron can exist for only a short time before decaying into a heavy, positively charged proton and a light, negatively charged electron by a process known as β-decay. However, a neutron can become stable by bonding to a proton. A suitable number of protons and neutrons bonded together form a positively charged nucleus. The traditional image of an atom is a nucleus surrounded by light, negatively charged electrons. Interestingly, a state in which electrons orbit a single nucleus is not necessarily the most stable state. This instability leads to the formation of molecules. Small molecules combine to form polymers, which are essential for living matter, and eventually leading, as the degree of complexity increases, to human beings.

Most interestingly, the neutron mass is only 0.08% greater than the sum of the masses of a proton and an electron. Consequently, the half-life of a neutron is 10.4 minutes. It is assumed that when the universe was created with the Big Bang, approximately the same number of protons and neutrons were created. Subsequently, neutrons started to decay into protons. However, the half-life of a neutron was sufficiently long for deuterons to form by neutrons bonding to protons before neutrons had disappeared. Helium was then formed. If the half-life of a neutron was any shorter, very little helium would have formed and heavier atoms would not have been created. Accordingly, intelligent life, which depends on complex molecules, would have never come into existence. On the other hand, if the half-life was longer, neutrons would be more stable and neutron stars would have readily formed. In this case, luminous stars would not have formed, making the conditions for the creation of intelligent life very difficult.

The instability range of the neutron that permits the birth of intelligent life is extremely narrow. It may be possible to explain the mechanism that determined the half-life of the neutron using laws and constants that are more fundamental. However, it seems that this leads to an endless cycle of searching for the origin of basic laws and constants. One might almost be inclined to concede that God himself selected the exact instability of the neutron. Whatever its origin, the instability of the neutron is a remarkable thing.

1.2. Nuclear Fission

As mentioned above, a nucleus consists of protons and neutrons, which are thus termed nucleons. Light atomic nuclei are highly symmetric and consist of approximately the same number of protons and neutrons. As the nucleus size increases, the Coulomb repulsion due to the positive charges of the protons makes the nucleus unstable. However, atomic nuclei with more neutrons than protons become stable again. The largest atomic nucleus that occurs naturally on earth is uranium-238 (^{238}U), which has 146 neutrons and 92 protons.

It is very difficult to cause nuclear reactions between atomic nuclei because the strong Coulomb repulsion due to the positive charge of nuclei hinders nuclei approaching each other. However, neutrons can readily cause nuclear reactions since they are neutral. A low-energy neutron is more likely to cause a nuclear reaction because of quantum effects. Neutron absorption usually occurs when a low-energy neutron collides with a nucleus. When a neutron is absorbed by a nucleus, the newly created nucleus generally gains excess energy. This excess energy increases the internal kinetic energy of the nucleus making it unstable. In most cases, the excess energy is eventually released as high-energy electromagnetic waves (γ rays) and the nucleus becomes stable. However, when a neutron collides with uranium-235 (^{235}U), the nucleus gains a large amount of excess energy, and since it consists of many nucleons, it starts vibrating like a liquid drop before eventually breaking into two nuclei of similar size with a very high probability. This nuclear reaction is called nuclear fission and the two generated nuclei are termed fission products.

In conventional nuclear reactors such as light-water reactors (LWRs), high-energy neutrons produced by fission collide with light nuclei, reducing the energy of the neutrons. By repeated collisions, the neutrons are rapidly moderated and they attain a final energy that is similar to the kinetic energy of the collision target (i.e., the thermal energy of the medium). Hence, moderated neutrons are termed thermal neutrons. Nuclear fission occurs if a thermal neutron is absorbed by ^{235}U. On the other hand, uranium-238 (^{238}U) does not undergo nuclear fission by thermal neutron absorption since a nucleus with even numbers of neutrons and protons is more stable than one with an odd number of either neutrons or protons. That is, ^{235}U contains 143 neutrons, which is odd, but after absorbing a neutron, it contains 144 neutrons, which is even. However, the neutron number of ^{238}U becomes odd after neutron absorption. Because of this difference, ^{235}U gains more excess energy than ^{238}U on neutron absorption, so that nuclear fission occurs for ^{235}U but it does not for ^{238}U. A nuclide that fissions after absorbing a thermal neutron is called fissile material, whereas a nuclide that does not fission but becomes a fissile material is called fertile material.

As mentioned above, a heavy nucleus has a higher ratio of neutrons to protons than a light nucleus. In nuclear fission, a heavy nucleus is converted into two nuclei each with approximately half the mass of the parent nucleus. Consequently, the number of neutrons exceeds that required for nuclear stability so that two to three neutrons are usually ejected per nuclear fission. The number of emitted neutrons, which is about 2.5 on average, is very important in the remainder of this book. It is less than the number of excess neutrons derived from the difference between the number of neutrons contained in ^{235}U plus one (144) and the total number of neutrons contained in the two fission product nuclei produced by ^{235}U fission. Not all the excess neutrons are released; the majority of them are retained in the fission products. These nuclei are unstable, but they gradually stabilize as the excess neutrons decay into protons. Only a small fraction of the nuclei are stabilized by releasing neutrons. Neutrons released in this way are called delayed neutrons. Delayed neutrons play an important role in the operation of a nuclear reactor; however, the explanation of this role is omitted here.

Even for stable nuclei, some nuclei are more stable than others. The peak of stability is located around iron; in other words, nuclei that are heavier and lighter than iron are less stable than iron. Uranium is located at the heaviest end. It is intrinsically unstable and changes very slowly into a lighter nucleus by successively releasing αand ß-particles. In nuclear fission, a very unstable nucleus is converted into two much more stable nuclei in a single reaction. Energy is released when an unstable state changes to a stable state. The energy released per nuclear fission is about 200 MeV (200×10^6 eV). In contrast, burning fossil fuel is a chemical reaction; the heat released per chemical reaction is measured in electron volts. Comparing the two reveals that the heat released by nuclear fission is extremely large.

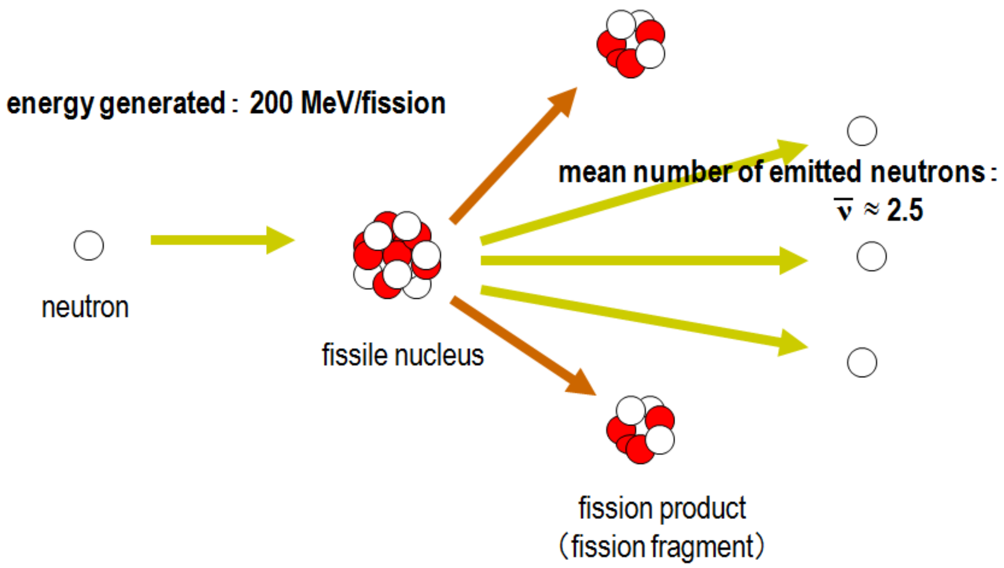


Figure 1-1. Characteristics of nuclear fission.

Figure 1-1 summarizes the characteristics of nuclear fission.

1.3. Chain Reactions and Criticality Control

As mentioned above, if a fissile material absorbs a neutron, nuclear fission occurs with a high probability with the release of two to three neutrons. The newly generated neutrons may then induce other nuclear fissions. A string of such successive nuclear fissions induced by the generated neutrons is called a chain reaction (see Figure 1-2).

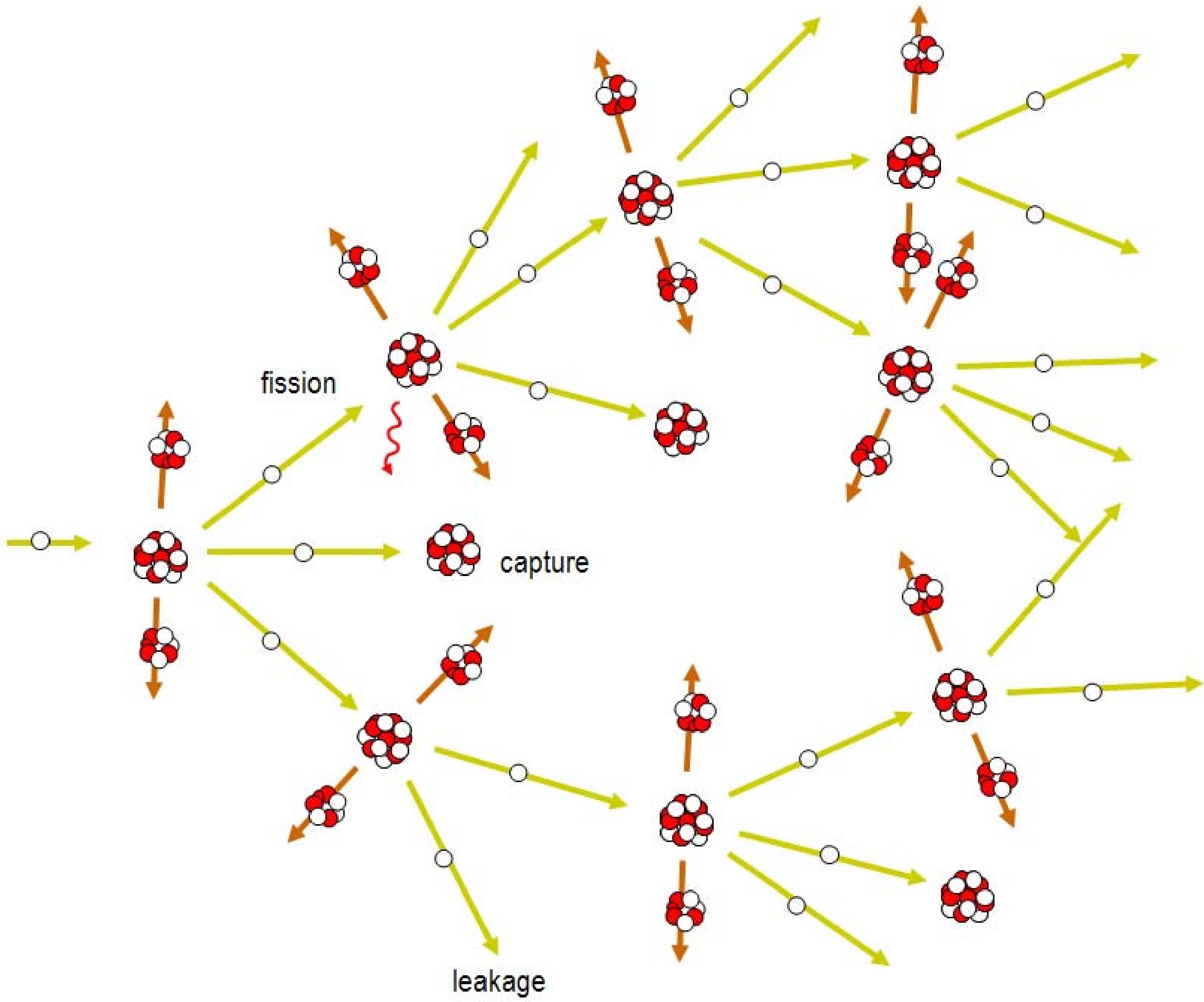


Figure 1-2. Nuclear fission chain reaction.

The number of neutrons in the system may increase or decrease with time or it may remain the same. This is a very important behavior for nuclear reactors and it is encapsulated by the neutron multiplication factor. A nuclear fission chain reaction progresses from one generation of nuclear fission to a succeeding generation. The neutron multiplication factor (usually denoted by *k*) is defined as the ratio of the number of neutrons in one generation to the number of neutrons in the preceding generation [Duderstadt, Hamilton, 1976]:

$$k \equiv \text{Neutron multiplication factor}$$

$$\equiv \frac{\text{Number of neutrons in one generation}}{\text{Number of neutrons in the preceding generation}} \tag{1-1}$$

When this value is equal to unity, the number of neutrons remains constant with time and the system is in what we term a critical state. When the value exceeds unity, the number of neutrons increases with time, giving rise to a supercritical state. When the value is less than unity, the number of neutrons decreases with time, giving rise to a subcritical state.

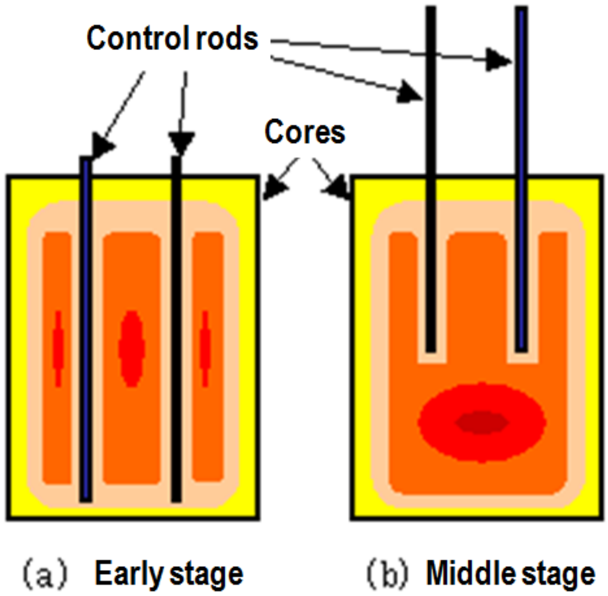


Figure 1-3. Power density change with the progression of burning in a conventional reactor.

In a nuclear reactor operating at a constant power, the number of neutrons is constant so that the neutron multiplication factor is unity. To terminate operation of the reactor, the neutron multiplication factor needs to be made sufficiently smaller than unity. This is accomplished by inserting a neutron absorber into the core (the fuel region of the nuclear reactor), as shown in Figure 1-3. In this way, neutrons generated by nuclear fission are absorbed by the neutron absorber, which causes the neutron multiplication factor to drop below unity. Neutron absorbers are usually rod shaped and thus they are referred to as control rods.

The operation conditions also affect the neutron multiplication factor. For example, changing the core temperature will alter the multiplication factor. It is highly problematic if the neutron multiplication factor increases with increasing temperature.

In this case, even if the initial state is critical (i.e., the neutron multiplication factor is unity), since nuclear fission increases the temperature, the number of nuclear fissions will increase (i.e., the neutron multiplication factor increases to above unity). This in turn will cause the temperature to increase further, resulting in a further increase in the number of nuclear fissions. In this way, the nuclear reactor will enter a vicious cycle that will result in a runaway reaction. Hence, it is essential to design nuclear reactors such that the neutron multiplication factor decreases with increasing temperature. In such reactors, when the temperature increases due to an increase in nuclear fission (i.e., when the neutron multiplication factor is greater than unity), the neutron multiplication factor will decrease and eventually converge to unity. In other words, the nuclear reactor counteracts external disturbances that raise the temperature thereby stabilizing operation.

The production, distribution, and consumption of neutrons are known as the neutron economy. The neutron multiplication factor is estimated from the rates of neutron reactions by

$$k \equiv \frac{\text{Rate of neutron production in reactor}}{\text{Rate of neutron loss (absorption plus leakage) in reactor}} \tag{1-2}$$

One important task for reactor engineers is to maximize the valuable utilizations of neutrons for the condition $k = 1$. A system is considered to have a better neutron economy, when it is possible to use more neutrons in valuable utilizations. CANDLE reactors require an excellent neutron economy, as mentioned later in this book.

Neutron production and absorption rates in a reactor depend on the materials in the reactor and the neutron leakage rate depends on the reactor geometry. Neutron production and absorption are more fundamental and important than neutron leakage when considering nuclear reactors since leakage can be reduced to negligible levels by simply increasing the reactor core size. Therefore, we often use the following neutron multiplication factor for the case when there is no neutron leakage; it is known as the infinite-medium neutron multiplication factor k_∞:

$$k_\infty \equiv \text{Infinite-medium neutron multiplication factor}$$

$$\equiv \frac{\text{Neutron production rate in reactor}}{\text{Neutron absorption rate in reactor}} \tag{1-3}$$

The previously introduced neutron multiplication factor is termed the effective neutron multiplication factor and is denoted by k_{eff}.

$$k_{eff} \equiv \text{Effective neutron multiplication factor}$$

$$\equiv \frac{\text{Neutron production rate in reactor}}{\text{Neutron loss (absorption plus leakage) rate in reactor}} \tag{1-4}$$

As the size of the reactor tends to infinity, k_{eff} approaches k_∞. This is why it is known as the infinite-medium neutron multiplication factor.

1.4. Burnup and Burning Control

Nuclear reactors differ from fossil fuel power plants in the way they use fuel. In a fossil fuel power plant, a large amount of fuel must be continuously supplied to the furnace; in contrast, once fuel has been inserted into a nuclear reactor, it can be kept in the reactor for years. Hence, a nuclear reactor has a high energy security; that is, it can continue to operate even when the fuel supply is suspended. Both nuclear reactors and fossil fuel power plants consume fuel. By analogy with fossil fuel power generation, the fuel consumption by a nuclear reactor is referred to as 'burnup'.

What happens as fuel burnup progresses in a nuclear reactor in a critical state? In widely operated LWRs, the amount of fissile material decreases and fission products accumulate. The reactor is initially put in a critical state by adjusting the neutron multiplication factor to be unity. However, the multiplication factor decreases to below unity with the progression of burning, so that the reactor will become subcritical if nothing is done. The following method is generally adopted to overcome this problem. Initially, an excess of neutron absorbers is placed in the reactor. As the neutron multiplication factor decreases due to the reduction in the fuel with burning, the amount of neutron absorber in the reactor is reduced so that the multiplication factor returns to unity. The neutron absorber can be reduced by either the operator withdrawing the neutron absorber, as described in the preceding section, or by using a neutron absorber that decreases with burning. In the latter case, a neutron absorber is selected that is converted to a material that has a lower neutron absorption when it absorbs neutrons. However, adjustment of the conversion rate is a critical design challenge. Such a neutron absorber is termed a burnable poison. It is difficult to maintain a nuclear reactor in an exact critical state using only burnable poison and it is necessary to include a human-operated control mechanism. Nevertheless, burnable poison considerably reduces the requirements for the human operator.

1.5. Neutron Economy, Use of Excess Neutrons

The only fissile material that occurs naturally is ^{235}U. It has a shorter half life than the other naturally occurring isotope ^{238}U. Both have been present in the earth since the earth was formed, but ^{235}U decays faster than ^{238}U and hence natural uranium presently contains only 0.7% ^{235}U, with the remainder being ^{238}U. Thus, many neutrons generated by nuclear fission in natural uranium are absorbed by ^{238}U, which prevents chain reactions from occurring. However, ^{235}U has a much higher reaction rate with thermal neutrons than ^{238}U. By applying the characteristics of nuclear fission, Fermi succeeded in constructing the first nuclear reactor. To moderate the neutrons, he mixed natural uranium with pure graphite in a heterogeneous structure. The reactor had to be huge to limit neutron leakage. This requirement for a large reactor indicates how difficult it is to achieve criticality and how few neutrons are available for use. The neutron economy is a key issue in this problem.

Recall that the number of neutrons generated by nuclear fission is two to three. However, a fissile nuclide does not always fission after the absorption of neutrons; it may remain a heavy nucleus after absorbing a neutron. Therefore, in the discussion of neutron economy including criticality and the effective use of neutrons, the number of generated neutrons per neutron absorption is more pertinent than the number of generated neutrons per nuclear fission. This value is called η (the Greek letter, read "eta").

Nuclides with high η values can realize a good neutron economy. As Figure 1-4 shows, the value of η varies with the nuclide and the absorbed neutron energy. The important neutron energy ranges in which most fission reactions occur are 10^{-2} – 1 eV for thermal reactors including Fermi's first reactor, pressurized water reactors (PWRs), boiling water reactors (BWRs), and high-temperature gas-cooled reactors (HTGRs), and 10 keV – 1 MeV (= 10^4 – 10^6 eV) for fast reactors. High energy neutrons are called as fast neturons, and reactors using fast neturons are called fast reactors. Plutonium-239 (^{239}Pu) has much higher values of η in the neutron energy range for fast reactors than ^{235}U and η of ^{239}Pu increases drastically with increasing neutron energy from about 10 keV.

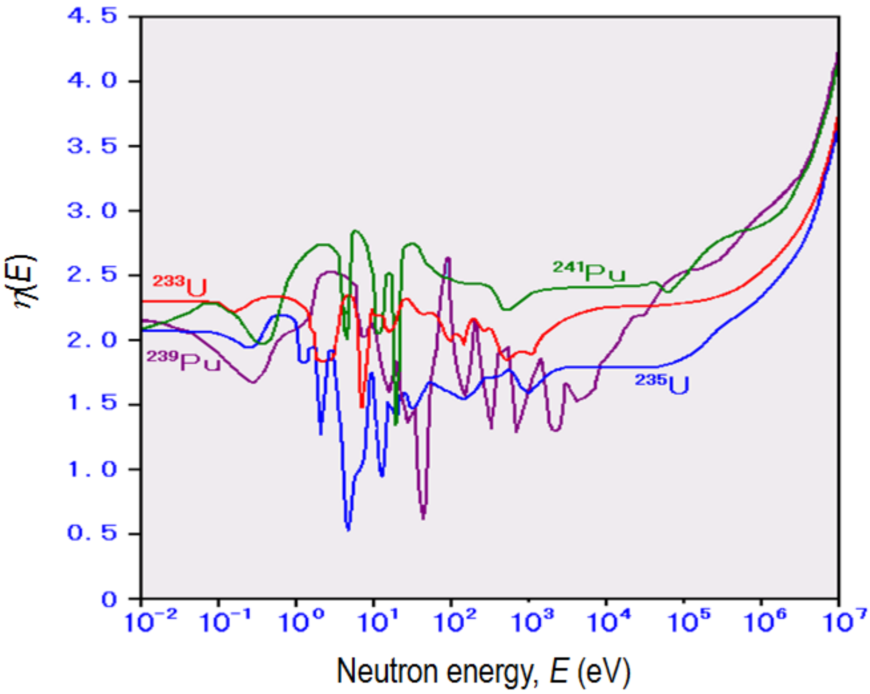


Figure 1-4. Values of η for typical fissile materials [Shibata et al., 2002].

If ^{238}U absorbs a neutron, it will become the fissile material ^{239}Pu; ^{238}U is thus a fertile material. Nuclear fission of ^{239}Pu induced by neutron absorption generates more neutrons than nuclear fission of ^{235}U, when high-energy neutrons are absorbed. Thus, ^{239}Pu can be generated by allowing ^{238}U to absorb excess neutrons, and criticality can be achieved in a nuclear reactor by a chain reaction of nuclear fission of mainly ^{239}Pu. In this way, more ^{239}Pu can be generated than is lost. If this can be achieved, natural uranium can be used for nuclear fission. (Not all natural uranium will be used since some plutonium will be mixed in the waste when recovering plutonium from spent fuel. This method can use approximately 70% of natural uranium. However, this is about 100 times greater than conventional methods using thermal neutrons for fission generation, which are able to use only about 0.7% of natural uranium.)

A nuclear reactor generates large quantities of radioactive material. If there are excess neutrons, it is possible to convert radioactive waste into harmless, stable material through nuclear reactions by using these neutrons. If neutrons are used for nuclear fission and the generation of fissile nuclides, the number of excess neutrons available for this purpose will be less than one per fission reaction. In reality, the amount of excess neutrons available for converting radioactive waste into harmless materials is limited when wasteful neutron absorption and leakage are considered. However, stabilizing radioactive waste generated by nuclear power would be a revolutionary breakthrough. How to achieve this goal is a very interesting challenge. One potential solution is CANDLE burnup.

2. What is the CANDLE Burnup Strategy?

2.1. Concept of the CANDLE Burnup Strategy

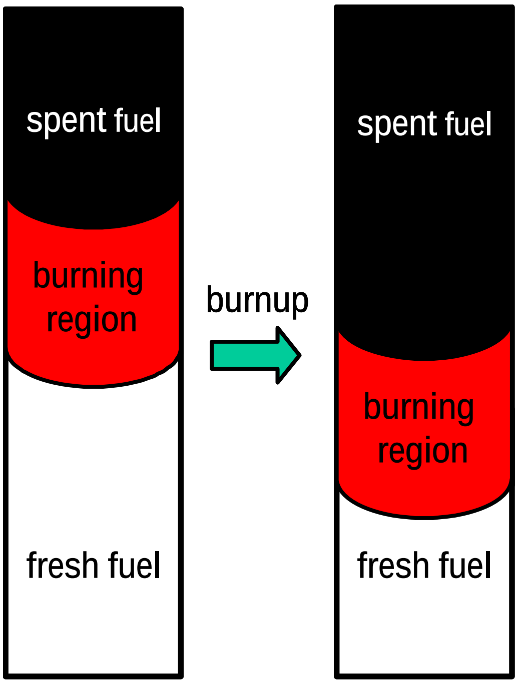


Figure 2-1. CANDLE burnup strategy. (Note that the direction of movement may be in the opposite direction to that illustrated and that the core is depicted as being extremely long to simplify the explanation.)

CANDLE stands for Constant Axial Shape of Neutron Flux, Nuclide Densities and Power Profile During Life of Energy Production [Sekimoto, Ryu, 2000a]. This acronym also implies candle-like burnup. As shown in Figure 2-1, when this burnup strategy is employed, the burning region propagates along the core axis at a speed proportional to the power output without changing the spatial distributions of the nuclide densities, neutron flux (speed-weighted average number density of neutrons), and power density. Significantly, unlike conventional reactor designs, it is not necessary to employ movable components to control burnup (e.g., control rods and reflector control), despite the fuel being fixed in the core. Note that the core is depicted as being extremely long in Figure 2-1 to clarify the burnup strategy characteristics. An infinitely long core is the simplest to treat mathematically. In an actual core, the combined length of the spent and fresh fuel regions is usually much shorter than the burning region. Figure 2-4 (presented later) shows a more realistic depiction of an actual reactor (although the movement distance is shown as being longer than it actually is). Note also that the burning region is depicted as propagating from top to bottom in Figure 2-1, but it is possible for it to move in the opposite direction.

CANDLE burning can occur in a core designed to have an infinite-medium neutron multiplication factor k_∞ (defined in Eq. (1-3)), so that k_∞ of the fuel varies with increasing neutron fluence in the manner shown in Figure 2-2. In the figure, k_∞ is plotted against the neutron fluence, which is obtained by integrating the neutron flux with respect to time and considered to be proportional to the burnup. k_∞ of fresh CANDLE fuel is less than unity, but it increases with increasing burnup, eventually exceeding unity. After reaching a maximum, it decreases, becoming less than unity again.

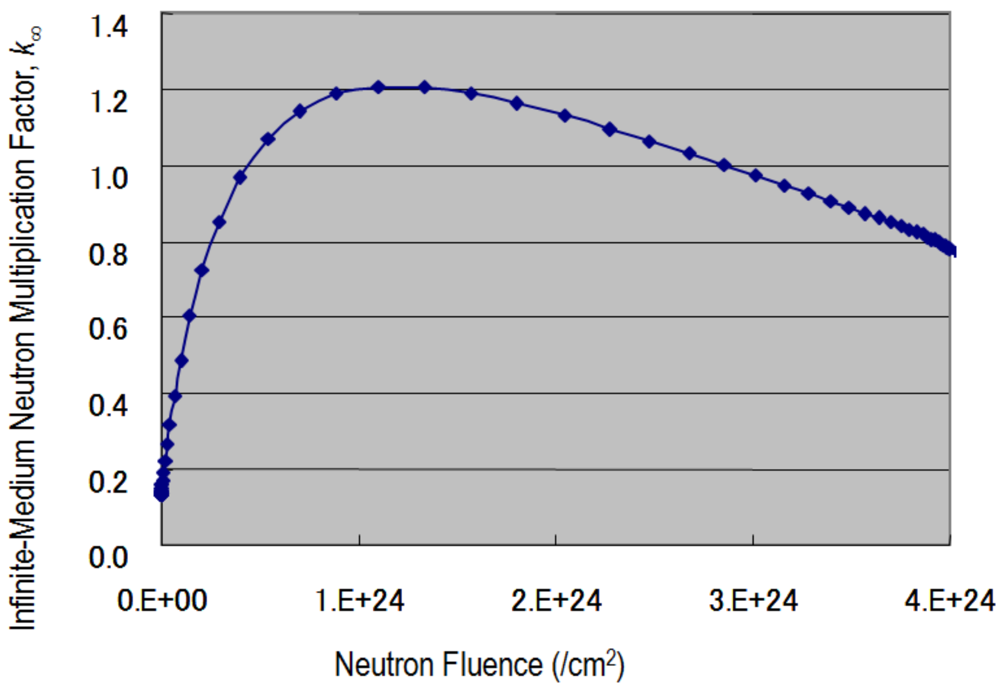


Figure 2-2. Infinite-medium neutron multiplication factor k_∞ as a function of neutron fluence.

Figure 2-3 plots the same data as that in Figure 2-2 with distance along the core axis (i.e., the z-axis) on the abscissa. Fresh fuel is on the left-hand side and spent fuel is on the right-hand side. k_∞ increases with burnup on the left-hand side of the peak and it decreases on the right side. Consequently, the peak is shifted to the left (i.e., to the fresh fuel side). The maximum neutron flux is located near the k_∞ peak. k_∞ is small far from the peak, becoming less than unity, and the neutron flux approaches zero. As a result, burnup does not occur and k_∞ is constant on the far left and right. In an equilibrium state, the spatial distribution of k_∞ does not vary with time; it only shifts to the fresh fuel side. It is not formidably difficult to generate the k_∞ variation depicted in Figure 2-2. Specific methods for achieving this vary depending on the nuclear reactor; they are explained later.

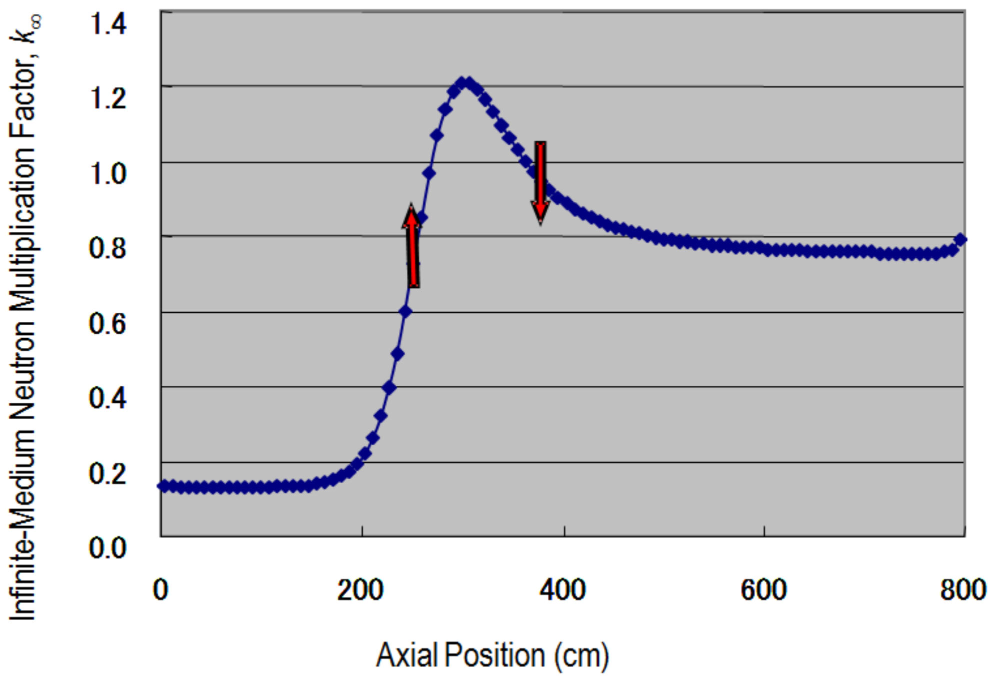


Figure 2-3. Infinite-medium neutron multiplication factor of fuel plotted against the central axis (z-axis). Arrows indicate the directions of change with burnup.

Even if the power level is varied, the shape of the power distribution remains constant; only the absolute values of the power distribution change. The burning region moving speed is proportional to the power level; the principle behind this is explained in Chapter 3, “Mathematical Explanation and Analysis Method”.

In reality, the core height is finite. The fuel should be exchanged by removing the spent fuel region and adding fresh fuel in the burnup direction when the burning region reaches the end of the core (see Figure 2-4). In this way, CANDLE burnup can be sustained.

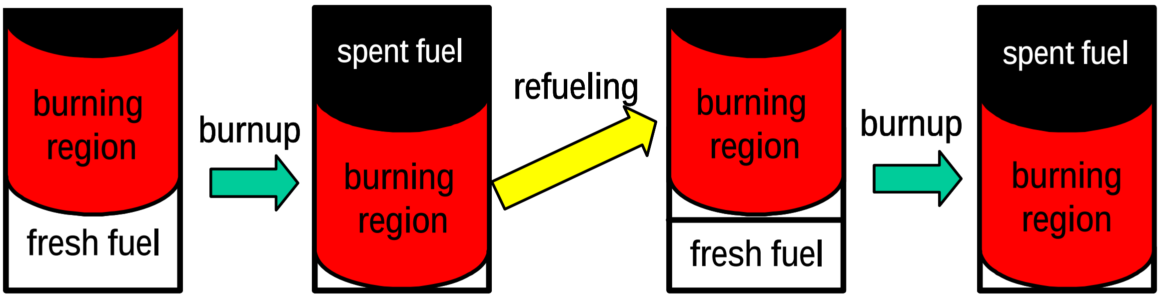


Figure 2-4. Refueling in the CANDLE burnup strategy.

Once an initial core has been prepared, it is straightforward to prepare subsequent cores. However, since short-lived radioactive materials cannot be used in the steady-state burning region of the initial core, it is difficult to fabricate this region from only easily obtainable materials. It may be necessary to use control rods when it is not possible to fabricate an ideal initial core so that burnup results in large variations in excess reactivity. In such a case, the best approach may be to construct a special reactor for the first several cores and install control rods to control the excess reactivity. After the first several cores have burned, fuel for the remaining cores is generated that has a composition close to that of an ideal CANDLE core. The new cores are then transferred to a conventional CANDLE reactor, which has no mechanism for controlling the excess reactivity. In this way, many initial CANDLE cores can be produced using one nuclear reactor.

The above has explained the principle of CANDLE burning in the equilibrium state. After the initial core has been prepared, CANDLE burning can be sustained indefinitely provided natural or depleted uranium is available. However, the method for preparing the initial core has not been explained. Even the initial core has a burning region that contains many unstable materials. Chapter 7 describes the method for preparing the initial core.

The following sections describe two examples of CANDLE burning: one for a fast reactor and the other for a thermal reactor.

2.1.1. Hard-Spectrum Fast Reactors

The energy distribution of neutrons is called neutron energy spectrum or simply neutron spectrum. When the average neutron energy of the spectrum are high, it is called hard spectrum. Since hard-spectrum fast reactors have excellent neutron economies, CANDLE burnup was tried for these reactors using natural or depleted uranium as fresh fuel [Sekimoto, Ryu, 2000a; Sekimoto et al., 2001a]. Figure 2-5 shows the variations in the nuclide densities of important nuclides with distance along the core axis when natural uranium is used as the fresh fuel. These nuclide density distributions realize the k_∞ profile shown in Figure 2-3.

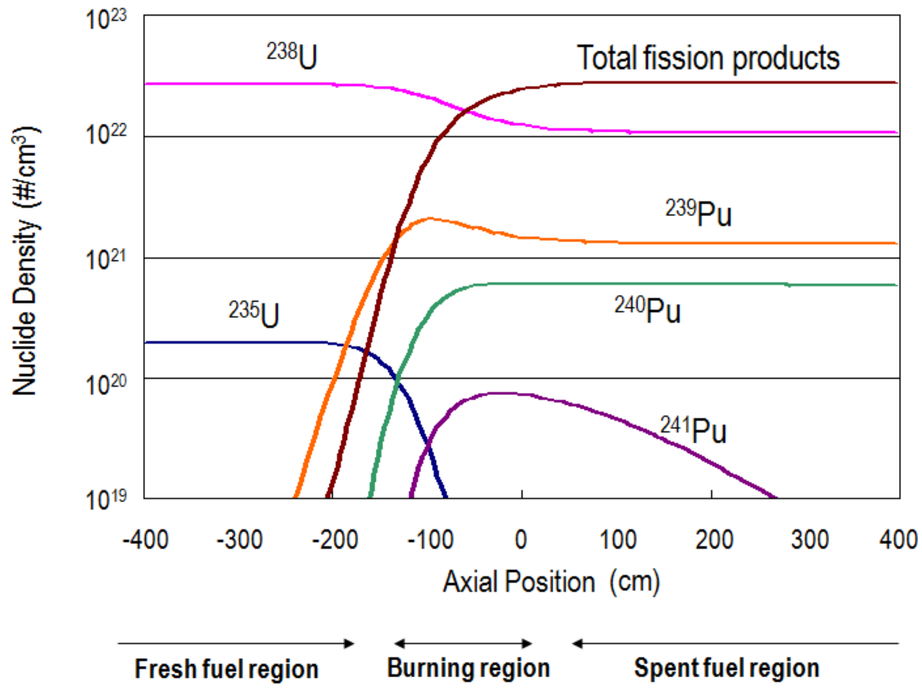


Figure 2-5. Nuclide number densities as a function of distance along core axis in a fast reactor.

^{238}U in the fresh fuel region absorbs neutrons leaking from the burning region and becomes ^{239}Pu. The ^{239}Pu density increases at the boundary between the fresh fuel region and the burning region and k_∞ at this position increases as burnup progresses. The ^{239}Pu density saturates in the burning region when the ^{239}Pu production rate equals its rate of destruction. In the same region the total fission product density continues to increase. Therefore, k_∞ at the boundary between the burning region and the spent fuel region decreases as burnup progresses.

Since natural uranium is highly subcritical, many neutrons need to be absorbed by ^{238}U to produce a large amount of ^{239}Pu to bring the system into a critical state. Thus, it is important to use a nuclear reactor that has an excellent neutron economy. For this purpose, the neutron spectrum should be extremely hard, as Figure 1-4 implies.

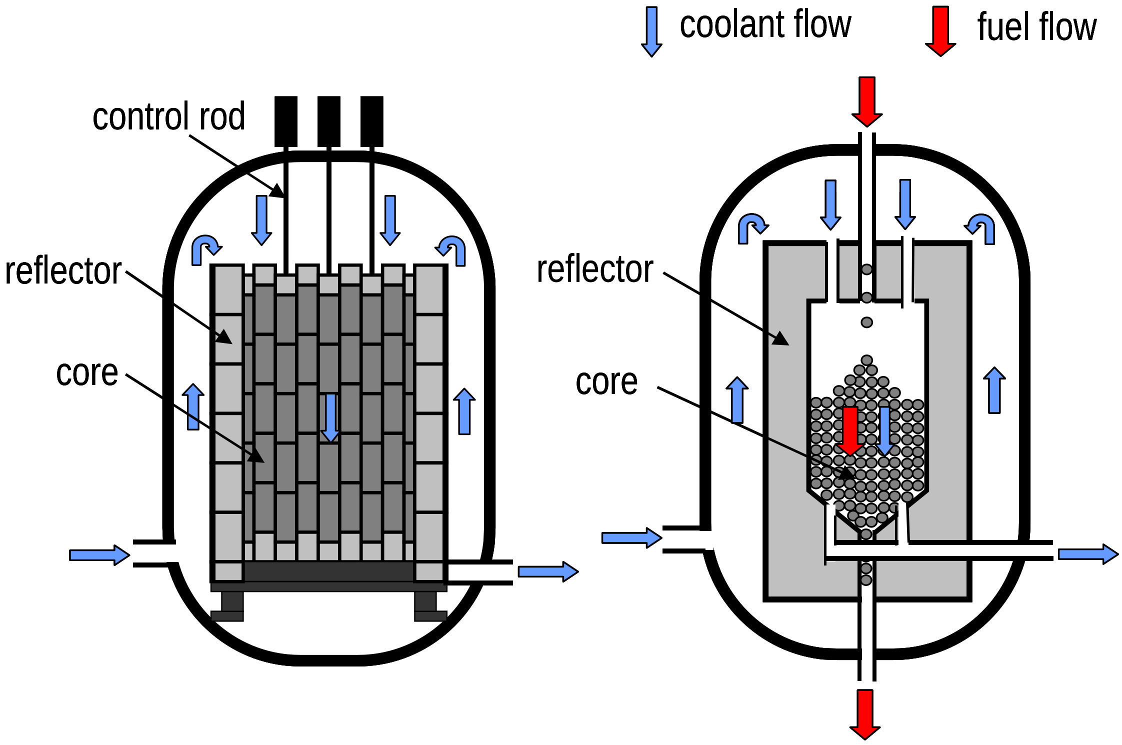
The fuel burnup is increased by supplying many neutrons to the fresh fuel region. This results in a high burnup of spent fuel and considerably reduces the burning region moving speed.

Edward Teller proposed a similar idea [Teller et al., 1996] using thorium. However, CANDLE burnup cannot be achieved in the truest sense when only thorium is used.

2.1.2. Thermal Reactors with Burnable Poison

HTGRs [Sekimoto et al., 2002] have been attracting growing interest and various applications are envisioned based on their use of high-temperature gas. Recently, the high safety and excellent nuclear economies of these reactors have been exciting interest and construction of commercial HTGRs is planned. A further advantage of these reactors is that the integrity of coated fuel particles in the reactor can be maintained up to a high burnup; thus, they are considered to be suitable reactors for eliminating plutonium and minor actinides.

HTGRs can be broadly classified into block-fuel and pebble-bed reactors. Figure 2-6 shows schematic diagrams of these two reactors. Note that the length ratios in these illustrations differ greatly from the actual length ratios; for example, the pebbles (fuel spheres) in the pebble-bed reactor are the size of tennis balls. The pressure vessels in these nuclear reactors are similar in size to those of large LWRs. The control rod driving mechanism is illustrated only for the block-fuel reactor. The pebble-bed reactor requires only a control rod driving mechanism for start–stop control; control rods are not necessary for burnup control. The pebble-bed reactor has the advantage that it can be refueled during operation, but it has some technical complications.



(a) Block-fuel HTGR (b) Pebble-bed HTGR

Figure 2-6. Schematic diagrams of HTGRs.

Of the currently operating nuclear reactors, block-fuel HTGRs are the most suitable for CANDLE burnup since they do not require any drastic design changes to introduce the fuel cycle scheme depicted in Figure 2-4 [Ohoka, Sekimoto, 2004a].

Figure 2-7 shows the variations in the nuclide densities of important nuclides and neutron flux with distance along the core axis for CANDLE burnup in a block-fuel HTGR. This figure indicates that the k_∞ profile along the core axis should be similar to that shown in Figure 2-3.

CANDLE burnup can be realized in a thermal reactor by adding burnable poison to the fuel. Gadolinium is employed as the burnable poison in Figure 2-7. When the reaction rate of the burnable poison is considerably greater than that of the fissile material, the burnable poison will absorb neutrons leaking from the burning region to the fresh fuel region and will quickly be consumed, as shown in Figure 2-7. In the figure, the ^{157}Gd density does not decrease to zero because ^{157}Gd is replenished by fission. Thus, fissile material starts to be active in the fresh fuel region and the burning region moves into this region, realizing CANDLE burnup. Burnable poison is currently used in conventional nuclear reactors to suppress excess reactivity during burnup. For this purpose, the neutron absorption rate is adjusted by introducing lumped geometry of burnable poison. However, in CANDLE burnup, the burnable poison should ideally disappear as soon as possible. Thus, it is mixed in diluted concentrations into a graphite matrix to increase neutron absorption rate.

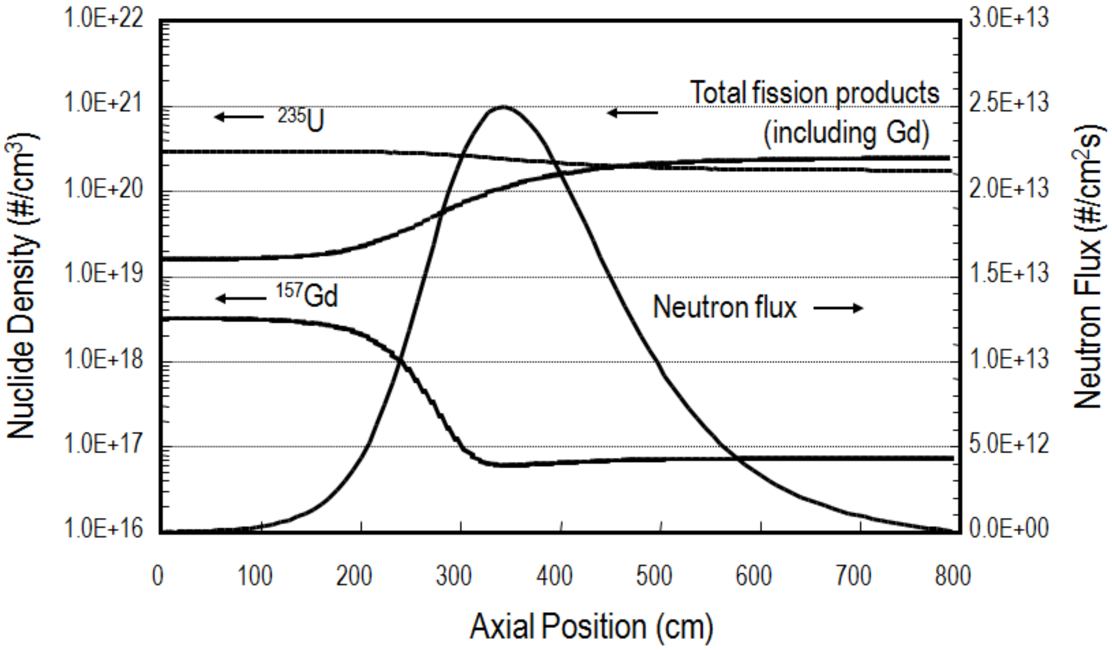


Figure 2-7. CANDLE burnup in a block-fuel HTGR.

CANDLE burnup can be realized in a thermal reactor by adding burnable poison to the fuel. Gadolinium is employed as the burnable poison in Figure 2-7. When the reaction rate of the burnable poison is considerably greater than that of the fissile material, the burnable poison will absorb neutrons leaking from the burning region to the fresh fuel region and will quickly be consumed, as shown in Figure 2-7. In the figure, the ^{157}Gd density does not decrease to zero because ^{157}Gd is replenished by fission. Thus, fissile material starts to be active in the fresh fuel region and the burning region moves into this region, realizing CANDLE burnup. Burnable poison is currently used in conventional nuclear reactors to suppress excess reactivity during burnup. For this purpose, the neutron absorption rate is adjusted by introducing lumped geometry

of burnable poison. However, in CANDLE burnup, the burnable poison should ideally disappear as soon as possible. Thus, it is mixed in diluted concentrations into a graphite matrix to increase neutron absorption rate.

Figures 2-4 and 2-6 show that, unlike pin-type fuel in LWRs, CANDLE-type refueling can be performed for block fuel without implementing any drastic design changes. After the reactor operation one block of spent fuel is removed and one block of fresh fuel is loaded. For the typical height of fuel block the lifetime of an operation cycle is usually a few years. Appendix A presents some examples and discusses this type of reactor.

2.2. Advantages of and Problems with the Burnup Strategy

CANDLE burnup shows very important characteristics that vary depending on the type of reactor. However, before describing these characteristics, general advantages of and problems with the CANDLE burnup strategy are explained.

Based on general considerations, it is expected to have the following advantages:

1) It does not require any control mechanism for burnup.

   In presently used general nuclear reactors, operation is continued for a fixed period between refuelings. As operation is continued with fuel in the core, fissile material is consumed and fission products (which waste neutrons) accumulate. Consequently, the criticality characteristics deteriorate. To lengthen the interval between refuelings, it is necessary to increase the amount of fissile materials and the positivity of the reactivity (defined as $(k-1)/k$, where k is the effective neutron multiplication factor of the core) after refueling. This causes the reactor to become supercritical, and it must be restored to criticality by inserting control rods. However, this wastes many neutrons. Furthermore, control rod malfunction and operational errors may result in serious accidents. In CANDLE burnup, control rods are not required to adjust the burnup reactivity, and hence it is expected to have the following advantages:

   - It does not waste neutrons. This is highly desirable since there are few excess neutrons, as mentioned in Chapter 1.

- Its operation is simple since it does not require burnup control.
- Inserting control rods into the core considerably distorts the power distribution, which varies greatly with the progress of burnup. This suppresses the average power density and reduces the neutron economy. These effects do not occur in CANDLE burnup.
- Accidents due to withdrawal errors of control rods cannot occur.
- Control rods kept continuously in a nuclear reactor lose their neutron absorption capability. CANDLE burnup does not require a countermeasure for this problem.

2) The core characteristics do not change as burnup progresses.

In a conventional nuclear reactor, the power density peaking factor and the power coefficient of reactivity vary as burnup progresses. Therefore, the control method needs to fully account for these effects. In CANDLE burnup, these parameters remain constant throughout burnup. As a result, operation does not change and it is very simple and reliable.

The calculation precision in reactor physics (of the criticality characteristics, power distribution, power coefficient of reactivity, etc.) is high. This is due not only to the high precision of the data and calculation methods used, but it is also the result of numerous criticality experiments. However, reactor physics calculations are difficult to verify experimentally when burnup has progressed, and the errors are large compared with those of fresh fuel calculations. This has necessitated including large safety margins in the power density peaking factors and power coefficient of reactivity due to burnup in conventional reactors. This type of consideration is less important for CANDLE burnup.

3) It is not necessary to adjust the flow rate with orifices as burnup progresses.

In conventional nuclear reactors, the power distribution in a plane perpendicular to the axis varies with the progress of burnup. Therefore, even if the coolant flow rate is adjusted at the beginning of burnup, so that the exit coolant temperature is constant (for flow parallel to the core axis), the flow rate changes as burnup progresses. If this change is too large, it is necessary to readjust the flow rate through the core coolant channel. For example, a long-life reactor has been proposed that employs an out–in burnup strategy in which the power peak moves from the exterior to the center. To optimize cooling of this reactor, the outside of the orifice is initially kept open and subsequently narrowed. At the center, it is initially narrowed and then opened. In CANDLE burnup, the axial integrated power distribution in the plane perpendicular to the axis does not change as burnup progresses. Therefore, it is not necessary to adjust the flow rate during burnup. Consequently, operation is easy and the probability of operational errors occurring is reduced.

4) High-level optimization of the radial power distribution is possible.

As mentioned above, the power distribution in conventional reactors varies in a complicated manner as burnup progresses. The optimum distribution at one point in time may deviate considerably from the optimum distribution at a later time. It is thus necessary to optimize the power distribution for the total core lifetime. In CANDLE burnup, once the power distribution has been optimized, it can be maintained throughout the core lifetime, enabling high-level optimization to be achieved. Some detail discussions on the radial power distribution flattening will be presented in Chapter 9.

5) The lifetime of a nuclear reactor can be easily lengthened by increasing the core height.

The core lifetime of a LWR can be lengthened by increasing the enrichment of the fresh fuel and increasing burnup. The lifetime is determined simply by the material integrity and allowable excess reactivity. To lengthen the lifetime beyond that attainable by this process, it is necessary to reduce the power density. Thus, even for the same burnup, the number of years of operation can be increased. To increase the core lifetime by a factor of M without altering the total power, the volume should be increased by a factor of M. On the other hand, to extend the lifetime of a CANDLE core, the core height should be lengthened, which increases the volume. If the burning region moves a distance D in the original design, then a length $(M-1)\times D$ should be added to the core height to increase the lifetime by a factor of *M*. In the power density strategy, the volume must be increased by a multiplicative factor, whereas in the CANDLE strategy, the volume need only be increased by an additive factor. Thus, the required volume increase in the CANDLE strategy is generally smaller than that in the power density strategy. The larger *M* is, the greater the difference between the two strategies will be. However, even for the CANDLE strategy, the required change in volume is expected to be large if *D* is large (although *D* is generally extremely small as shown in Chapter 4). CANDLE burnup has the following advantages because of this assumption:

- The burning region moving speed is generally very slow, making it is easy to design a super-long-life reactor.
- The core life can be easily altered by changing the core height.
- Once a small long-life reactor has been realized, a nuclear reactor can be constructed at a factory, transferred, and installed at the site, operated for a long time without changing fuel, and transferred back to the factory (for replacement with a new nuclear reactor). This is expected to give the following additional advantages:
    - Refueling is the most difficult of the normal operations of a nuclear reactor. Hence, the ability to refuel at the centralized dedicated factory is a big advantage when a reactor is operated at a location where high-level technology is not available.
    - A nuclear reactor in which fuel is semipermanently enclosed in its core has a high nuclear proliferation resistance.

6) k_∞ of fresh fuel in an exchanged core is less than unity.

As shown in Figure 2-2, an important feature of CANDLE burnup is that k_∞ of fresh fuel is less than unity. From a safety viewpoint, it is highly desirable for k_∞ of fresh fuel to be less than unity. This ensures that it has a very small possibility of becoming critical even when a large amount of fresh fuel is gathered together, thus making transportation and storage of fresh fuel simple and safe.

On the other hand, CANDLE burnup suffers from the following problems:

1) The core tends to be axially long and the coolant pressure loss tends to be large.
To realize a long-life reactor, a long core should be fabricated. This requires that the coolant channel length also be long. Consequently, there will be a large pressure loss, making it necessary to use a powerful pump.
However, this is not a problem provided the core is not extremely long. The length of burning region is not long as shown in Chapter 9. If the burning region moving speed is very slow, a long lifetime can be achieved without using a very long core. For example, the moving speed in a large fast reactor is typically about 4 cm/year (i.e., 40 cm in 10 years or 80 cm in 20 years), as shown in Chapter 4. These lengths are sufficiently short that even a conventional pump can cope with the increased pressure loss.

2) Limited ability to adjust the axial power density distribution.
An axial power distribution is inherent to CANDLE burnup. However, the radial power distribution can be optimized, as mentioned in advantage 4 above. The total power distribution is generally considered to be quite good. The radial power distribution optimization is discussed in Chapter 9.

3) It is difficult to prepare the initial core.
Preparation of exchange fuel is simple. However, fuel for the initial core must be prepared that can effectively stimulate the burning region. Since the burning region contains a considerable amount of radioactive material, it is difficult to stimulate it using readily available materials. The requirements are as follows:
- The effective neutron multiplication factor of the core in an equilibrium state should be unity.
- The effective neutron multiplication factor should vary little until the core reaches an equilibrium state.
- The CANDLE core should reach equilibrium rapidly.

It may be necessary to install control equipment if the effective neutron multiplication factor of the initial core varies greatly. As mentioned in Section 2.1, one solution for this problem is to construct a special nuclear reactor for preparing the fuel for the equilibrium core.

Chapter 7 discusses initial core preparation and a good example of it is given.

Various solutions have been proposed to address these problems. They are described in later chapters along with detailed results for several reactor designs.

3. Mathematical Explanation and Analysis Method

Some principles of the CANDLE burning are easy to understand when they are mathematically explained. In fact, the explanation of the analysis method is difficult without using some equations. In the chapter the mathematical explanation and analysis method for the CANDLE burning are described. I require the readers some introductory knowledge on reactor physics [Duderstadt, Hamilton, 1976]. Readers who do not like mathematics can skip this chapter since the other chapters can be understood without reading this chapter.

3.1. Basic Equations of Neutron Transport and Fuel Burning

For the investigation of CANDLE burning, we have to treat both neutron flux, $\phi(\vec{r},\vec{\Omega},E,t)$, and nuclide number density distributions, $N_n(\vec{r},t)$, as dependent variables. The variables t and $\vec{r}$ are time and position in three dimensional space, respectively, and E and $\vec{\Omega}$ are energy and direction of motion of neutron, respectively. The neutron flux satisfies the following neutron transport equation:

$$\begin{aligned}
&\frac{1}{v}\frac{\partial}{\partial t}\phi(\vec{r},\vec{\Omega},E,t)+\vec{\Omega}\cdot\nabla\phi(\vec{r},\vec{\Omega},E,t)+\sum_n N_n(\vec{r},t)\sigma_{T,n}(E)\phi(\vec{r},\vec{\Omega},E,t) \\
&\quad=\int_0^\infty dE'\int_{4\pi}d\vec{\Omega}'\sum_n N_n(\vec{r},t)\sigma_{S,n}(\vec{\Omega}'\cdot\vec{\Omega},E'\rightarrow E)\phi(\vec{r},\vec{\Omega}',E',t) \\
&\qquad+\frac{1}{4\pi}\int_0^\infty dE'\int_{4\pi}d\vec{\Omega}'\sum_n N_n(\vec{r},t)\chi(E)\nu_n(1-\beta_n)\sigma_{F,n}(E')\phi(\vec{r},\vec{\Omega}',E',t) \\
&\qquad+\sum_j\frac{\chi_j(E)}{4\pi}\lambda_j C_j(\vec{r},t) \\
&\qquad+S(\vec{r},\vec{\Omega},E,t)
\end{aligned}\tag{3-1}$$

where

$\sigma_{T,n}(E)$: microscopic total cross section of nuclide n for the neutron with energy E,

$\sigma_{S,n}(\vec{\Omega}'\cdot\vec{\Omega},E'\rightarrow E)$: microscopic scattering cross section of the nuclide n for the neutron with energy E' and direction $\vec{\Omega}$' transferred to E and $\vec{\Omega}$. This reaction includes any neutron emitting reactions except fission,

$\sigma_{F,n}(E)$: microscopic fission cross section of nuclide *n* for the neutron with energy *E*,

ν_n : average number of neutrons emitted by the fission of nuclide *n*,

$\chi(E)$: prompt fission neutron energy spectrum,

$\chi_j(E)$: delayed neutron energy spectrum for precursor *j*,

β_n : delayed neutron precursor fraction produced by the fission of nuclide *n*,

λ_j : decay constant of delayed neutron precursor *j*,

$C_j(\vec{r},t)$: number density of delayed neutron precursor *j*,

$S(\vec{r},\vec{\Omega},E,t)$: external neutron source.

Though $\chi(E)\nu_n(1-\beta_n)$ is a function of the energy of impinging neutron which causes the fission, the expression (E') in the equation is omitted for simplicity.

The delayed neutron precursor density $C_j(\vec{r},t)$ satisfies the following equation:

$$\frac{\partial}{\partial t}C_j(\vec{r},t)+\lambda_j C_j(\vec{r},t)=\int_0^\infty dE'\int_{4\pi}d\vec{\Omega}'\sum_n N_n(\vec{r},t)\beta_{n,j}\nu_n\sigma_{F,n}(E')\phi(\vec{r},\vec{\Omega}',E',t) \tag{3-2}$$

where

$\beta_{n,j}$: fraction of delayed neutron precursor *j* produced by the fission of nuclide *n*, which satisfies

$$\beta_n=\sum_j \beta_{n,j} \tag{3-3}$$

The nuclide density satisfies the following equation:

$$\frac{\partial}{\partial t}N_n(\vec{r},t)=-N_n(\vec{r},t)\left(\lambda_n+\int_0^\infty dE'\sigma_{A,n}(E')\int_{4\pi}\phi(\vec{r},\vec{\Omega}',E',t)d\vec{\Omega}'\right)$$
$$+\sum_{n'}N_{n'}(\vec{r},t)\lambda_{n'\to n}+\sum_{n'}N_{n'}(\vec{r},t)\int_0^\infty dE'\sigma_{n'\to n}(E')\int_{4\pi}\phi(\vec{r},\vec{\Omega}',E',t)d\vec{\Omega}' \tag{3-4}$$

where

$\sigma_{A,n}(E)$: microscopic absorption cross section of nuclide *n* for the neutron with energy E,

$\sigma_{n\to n'}(E)$: microscopic neutron cross section of nuclide n to be transformed to nuclide n',

λ_n : decay constant of nuclide n,

$\lambda_{n\to n'}$: decay constant of nuclide n to be transformed to n'.

Here the nuclide n covers both fuel and fission products. The production of fission product from fission can also be treated by the Eq. (4) by choosing $\sigma_{n\to n'}(E)$ properly.

In addition to these equations we consider the equations of temperature and motion of materials. Coolant flow distribution affects strongly temperature distributions, and the temperature distributions affect microscopic cross sections and material densities and shapes.

3.2. Steady State CANDLE Burnup Equation

In the following we consider the CANDLE burnup, in which the burning region moves at a speed proportionate to the power output along the direction of the core axis without changing the spatial distributions of the nuclide densities, neutron flux, or power density. In this case the reactor is in a critical state and the external neutron source does not stay in the core. Only time dependent phenomena considered in this article is the burnup. In this situation the time derivative of neutron flux in Eq. (3-1) and the time derivative of delayed neutron precursor density in Eq. (3-2) can be neglected. These two equations can be combined to the following equation:

$$\begin{aligned}
&\vec{\Omega}\cdot\nabla\phi(\vec{r},\vec{\Omega},E,t)+\sum_n N_n(\vec{r},t)\sigma_{T,n}(E)\phi(\vec{r},\vec{\Omega},E,t)\\
&\quad=\int_0^\infty dE'\int_{4\pi}d\vec{\Omega}'\sum_n N_n(\vec{r},t)\sigma_{S,n}(\vec{\Omega}'\cdot\vec{\Omega},E'\to E)\phi(\vec{r},\vec{\Omega}',E',t)\\
&\qquad+\frac{1}{4\pi k_{eff}}\int_0^\infty dE'\int_{4\pi}d\vec{\Omega}'\sum_n N_n(\vec{r},t)\chi_n(E)\nu_n\sigma_{F,n}(E')\phi(\vec{r},\vec{\Omega}',E',t)
\end{aligned}\qquad(3\text{-}5)$$

Here the reactor is usually kept critical by adjusting the control rods. However, it is complicated to implement this procedure in neutron transport equation, and introduce the effective neutron multiplication factor, k_{eff}, instead. The reactor is controlled to produce its total power output to equal to the aimed value, $P(t)$:

$$\int_V d\vec{r}\int_0^\infty dE'\int_{4\pi} d\vec{\Omega}'\sum_n N_n(\vec{r},t)\sigma_{F,n}(E')\phi(\vec{r},\vec{\Omega}',E',t) = P(t) \tag{3-6}$$

where $\int_V \bullet\, d\vec{r}$ denotes the integration over the whole core.

The basic equations for our study are Eqs. (3-4), (3-5) and (3-6).

We consider the ideal CANDLE burnup, where the burning region moves along the core axis without any change of flux and nuclide number density distributions. This ideal burnup can be realized for the core geometry, such that the core length should be infinite and the geometry perpendicular to the core axis is same for different axial position. We also consider the constant power operation and Eq. (3-6) becomes

$$\int_V d\vec{r}\int_0^\infty dE'\int_{4\pi} d\vec{\Omega}'\sum_n N_n(\vec{r},t)\sigma_{F,n}(E')\phi(\vec{r},\vec{\Omega}',E',t) = P_0\,. \tag{3-7}$$

In this scenario the neutron flux shape and nuclide densities move with burnup, whose relative shapes are not changed and their positions move with a constant speed V along z-axis for CANDLE burnup. For this burnup scheme, when the following Galilean transformation [Goldstein, 1950] given by

$$\vec{r}_G = \vec{r} + \vec{v}t \qquad (x_G = x\,,\ y_G = y\,,\ z_G = z + vt) \tag{3-8}$$

$$t_G = t \qquad , \tag{3-9}$$

is applied to Eqs. (3-4), (3-5) and (3-7), then they will be changed to

$$\begin{aligned}\left(v\frac{\partial}{\partial z_G} + \frac{\partial}{\partial t_G}\right)&N_n(\vec{r}_G,t_G)\\ &= -N_n(\vec{r}_G,t_G)\left(\lambda_n + \int_0^\infty dE'\sigma_{A,n}(E')\int_{4\pi}\phi(\vec{r}_G,\vec{\Omega}',E',t_G)d\vec{\Omega}'\right) + \sum_{n'} N_{n'}(\vec{r}_G,t_G)\lambda_{n'\to n}\\ &\quad + \sum_{n'} N_{n'}(\vec{r}_G,t_G)\int_0^\infty dE'\sigma_{n'\to n}(E')\int_{4\pi}\phi(\vec{r}_G,\vec{\Omega}',E',t_G)d\vec{\Omega}'\end{aligned} \tag{3-10}$$

$$\vec{\Omega}\cdot\nabla\phi(\vec{r}_G,\vec{\Omega},E,t_G)+\sum_n N_n(\vec{r}_G,t_G)\sigma_{T,n}(E)\phi(\vec{r}_G,\vec{\Omega},E,t_G)$$
$$=\int_0^\infty dE'\int_{4\pi}d\vec{\Omega}'\sum_n N_n(\vec{r}_G,t_G)\sigma_{S,n}(\vec{\Omega}'\cdot\vec{\Omega},E'\to E)\phi(\vec{r}_G,\vec{\Omega}',E',t_G)$$
$$+\frac{1}{4\pi k_{eff}}\int_0^\infty dE'\int_{4\pi}d\vec{\Omega}'\sum_n N_n(\vec{r}_G,t_G)\chi_n(E)\nu_n\sigma_{F,n}(E')\phi(\vec{r}_G,\vec{\Omega}',E',t_G)$$
(3-11)

and

$$\int_V d\vec{r}_G\int_0^\infty dE'\int_{4\pi}d\vec{\Omega}'\sum_n N_n(\vec{r}_G,t_G)\sigma_{F,n}(E')\phi(\vec{r}_G,\vec{\Omega}',E',t_G)=P_0 \tag{3-12}$$

If the speed of the transformed ordinate, v, is the same as the speed of burning region in CANDLE burnup, V, Eq. (3-10) becomes

$$V\frac{\partial}{\partial z_G}N_n(\vec{r}_G,t_G)$$
$$=-N_n(\vec{r}_G,t_G)\left(\lambda_n+\int_0^\infty dE'\sigma_{A,n}(E')\int_{4\pi}\phi(\vec{r}_G,\vec{\Omega}',E',t_G)d\vec{\Omega}'\right)+\sum_{n'}N_{n'}(\vec{r}_G,t_G)\lambda_{n'\to n}$$
$$+\sum_{n'}N_{n'}(\vec{r}_G,t_G)\int_0^\infty dE'\sigma_{n'\to n}(E')\int_{4\pi}\phi(\vec{r}_G,\vec{\Omega}',E',t_G)d\vec{\Omega}' \tag{3-13}$$

Equations (3-13), (3-11) and (3-12) are now time independent and we omit time variable in these equations. In the followings we will discuss on the Galilean transformed equations, and omit the suffix G for simplicity. Then Eqs. (3-13), (3-11) and (3-12) are rewritten as

$$V\frac{\partial}{\partial z}N_n(\vec{r})=-N_n(\vec{r})\left(\lambda_n+\int_0^\infty\sigma_{A,n}(E')\phi(\vec{r},E')dE'\right)$$
$$+\sum_{n'}N_{n'}(\vec{r})\lambda_{n'\to n}+\sum_{n'}N_{n'}(\vec{r})\int_0^\infty\sigma_{n'\to n}(E')\phi(\vec{r},E')dE' \tag{3-14}$$

$$\vec{\Omega}\cdot\nabla\phi(\vec{r},\vec{\Omega},E)+\sum_n N_n(\vec{r})\sigma_{T,n}(E)\phi(\vec{r},\vec{\Omega},E)$$
$$=\int_0^\infty dE'\int_{4\pi}d\vec{\Omega}'\sum_n N_n(\vec{r})\sigma_{S,n}(\vec{\Omega}'\cdot\vec{\Omega},E'\to E)\phi(\vec{r},\vec{\Omega}',E')$$

$$+\frac{1}{4\pi k_{eff}}\sum_{n} N_n(\vec{r})\chi_n(E)\nu_n\int_0^\infty \sigma_{F,n}(E')\phi(\vec{r},E')dE' \tag{3-15}$$

and

$$\int_V d\vec{r}\sum_n N_n(\vec{r})\int_0^\infty \sigma_{F,n}(E')\phi(\vec{r},E')dE' = P_0 \tag{3-16}$$

where we have used the scalar flux, $\phi(\vec{r},E)$, instead of angular flux, $\phi(\vec{r},\vec{\Omega},E)$:

$$\phi(\vec{r},E) \equiv \int_{4\pi}\phi(\vec{r},\vec{\Omega}',E)d\vec{\Omega}' \tag{3-17}$$

Generally we can ignore radioactive decay in Eq. (3-14), if we choose the nuclides properly. Usually the decay of ^{241}Pu and some fission products may contribute some, but their contribution to the effective neutron multiplication factor is almost negligible. In this case each nuclide density can be expressed by using neutron fluence, θ. Eq. (3-14) can be also written in good accuracy as

$$V\frac{\partial}{\partial z}N_n(\vec{r}) = -N_n(\vec{r})\int_0^\infty \sigma_{A,n}(E')\phi(\vec{r},E')dE' + \sum_{n'} N_{n'}(\vec{r})\int_0^\infty \sigma_{n'\to n}(E')\phi(\vec{r},E')dE' \tag{3-18}$$

If $\phi(\vec{r},\vec{\Omega},E)$ and $N_n(\vec{r})$ are the solution of Eqs. (3-15) through (3-18), $\alpha\phi(\vec{r},\vec{\Omega},E)$ and $N_n(\vec{r})$ are also the solution of Eqs. (3-15) through (3-18) for αV instead of V of Eq. (3-18) and for αP_0 instead of P_0 of Eq. (3-16). It means that if the power rate is changed, the relative power shape does not change but only power level and burning region speed change by that rate.

3.3. Approximated Treatment

In this section we try to derive simple functions of flux profile by simplifying Eqs. (3-15) through (3-18) and some important characteristics of the CANDLE reactor. They may be helpful for understanding CANDLE reactor. For simplicity the diffusion approximation is introduced to Eq. (3-15) and one-dimensional one-group treatment is considered, and after employing some assumptions the following equations are derived:

$$-D\frac{d^2}{dz^2}\phi(z)+\Sigma_a\phi(z)=\frac{k_\infty(\theta(z))}{k_{eff}}\Sigma_a\phi(z) \tag{3-19}$$

$$\phi(z)=V\frac{d\theta(z)}{dz} \tag{3-20}$$

where $\theta(z)$ is the neutron fluence at the axial position z.

By employing the diffusion length L, defind by

$$L^2=\frac{D}{\Sigma_a} \tag{3-21}$$

which has the physical interpretation as being $1/\sqrt{6}$ of the root mean square distance traveled by a neutron from its birth in fission to its eventual demise via absorption, Eq. (3-19) can be rewritten as

$$-L^2\frac{d^2}{dz^2}\phi(z)+\phi(z)=\frac{k_\infty(\theta(z))}{k_{eff}}\phi(z) \tag{3-22}$$

From the criticality condition

$$k_{eff}=1 \tag{3-23}$$

Then Eq. (3-22) becomes

$$-L^2\frac{d^2}{dz^2}\phi(z)+\phi(z)=k_\infty(\theta(z))\phi(z) \tag{3-24}$$

Here the infinite neutron multiplication factor k_∞ should satisfy the following conditions:

1. The curve of $k_\infty(\theta)$ is given for $0 \le \theta \le \theta_M$ and continuous.
2. Its value is less than 1 at $\theta = 0$, increases with increasing θ smoothly and monotonically until it takes the maximum value, and then decreases smoothly and monotonically until $\theta = \theta_M$, where it takes a value less than 1.
3. Its maximum value locates at about middle of this range.

We assume $k_\infty(\theta)$ as a simple function satisfying the above requirements as follows:

$$k_\infty(\theta) = -4(k_{\max} - k_{\min})\left(\frac{\theta}{\theta_M}\right)^2 + 4(k_{\max} - k_{\min})\left(\frac{\theta}{\theta_M}\right) + k_{\min} \tag{3-25}$$

By substituting Eqs. (3-20) and (3-25) into Eq. (3-24) and manipulating we obtain the following equation;

$$-L^2 \frac{d^3\theta}{dz^3} + \left(4(k_{\max} - k_{\min})\left(\frac{\theta}{\theta_M}\right)^2 - 4(k_{\max} - k_{\min})\left(\frac{\theta}{\theta_M}\right) + 1 - k_{\min}\right)\frac{d\theta}{dz} = 0 \tag{3-26}$$

The solution of this equation is given by [van Dam, 2000; Chen, Maschek, 2005]

$$\theta(z) = \theta_M \frac{1}{1 + e^{-\alpha z}} \tag{3-27}$$

where the axial position is chosen so that the flux peak position locates at $z = 0$. The flux is given by

$$\phi(z) = V\theta_M \alpha \frac{e^{\alpha z}}{\left(1 + e^{\alpha z}\right)^2} = V\theta_M \alpha \frac{1}{\left(1 + e^{-\alpha z}\right)\left(1 + e^{\alpha z}\right)} \tag{3-28}$$

where α is related to the half width, *W*, of flux shape

$$W = \frac{2\ln\left(3 + 2\sqrt{2}\right)}{\alpha} = \frac{3.5255}{\alpha} \tag{3-29}$$

and satisfies the following relations:

$$k_{\max} = 1 + \frac{1}{2} L^2 \alpha^2 \tag{3-30}$$

and

$$k_{\min} = 1 - \frac{1}{2} L^2 \alpha^2 \tag{3-31}$$

From Eq. (3-30) and (3-31)

$$k_{\max} + k_{\min} = 2 \tag{3-32}$$

and

$$k_{\max} - k_{\min} = L^2 \alpha^2 \tag{3-33}$$

The flux shape is characterized by *W*. *W* is given from Eqs. (3-29) and (3-30) as follows:

$$W = \frac{2.49L}{\sqrt{k_{\max} - 1}} \tag{3-34}$$

From this equation *W* is decreased by decreasing *L* and/or increasing $k_{\max}$, which is equivalent to decreasing $k_{\min}$ from Eq. (3-32). Therefore, the natural uranium or depleted uranium realizes shorter reactor core than conventional enriched fuels. Of course obtaining critical system using such an initial fuel is very difficult. It offers reactor core designers a challenging problem. Usually *L* is small enough, but $k_{\max}$ is nearly unity. Therefore, it is difficult to make *W* small and to design a compact core.

The maximum value of flux $\phi(z)$, which is presented by Eq. (3-28), is given by the following:

$$\phi_{\max} = \frac{1}{4} V \theta_M \alpha \tag{3-35}$$

Therefore the velocity of burning region is determined as

$$V = \frac{4\phi_{\max}}{\theta_M \alpha} = 1.13 \frac{\phi_{\max}}{\theta_M} W \tag{3-36}$$

It is already mentioned at the end of Section 3.2 that V is proportional to $\phi_{\max}$. This equation is easy to be understood from physical intuition, that both $\theta_M / \phi_{\max}$ and W / V are proportional to the time required for the burning region to pass the effective core height. Usually the burnup of spent fuel for CANDLE reactor is very high. It means that the value of θ_M is also very large, therefore V becomes very small from this equation.

3.4. Numerical Analysis of Steady State CANDLE burning

The simulation calculation of CANDLE burning is equivalent to solving Eqs. (3-1) through (3-4) simultaneously. The method for this calculation is already established. Therefore, I will not mention about it. You can find many calculation systems for many kinds of usages from extensive one to simple one. From them you can choose one proper to your requirement. If you want to optimize a reactor design and want to obtain accurate results for the design, CANDLE burning calculation takes so much time. It may be better for you to choose a simple calculation system employing many approximations which can perform a set of calculations in short period, though its accuracy becomes poorer.

If the design of CANDLE reactor is not fixed and tried to be made, generally it is better to solve the equilibrium steady state given by the set of Eqs. (3-14) through (3-17). This is not a familiar set, but each equation is familiar and you can find a proper method to solve them easily. Eq. (3-14) is a first-order differential equation, and several numerical methods are discussed in Ref. [Tachihara, Sekimoto, 1999]. Eq. (3-15) is very familiar to reactor physics, and many textbooks are available in this area [Clark, Hansen, 1964; Duderstadt, Hamilton, 1976; Ronen, 1986; Dupree, Fraley, 2002]. Here I will not mention how to solve these equations in detail, but mention about how to treat the coupling of these equations, since the latter method is unique to the analysis of CANDLE burning.

In this section I will show the method to solve Eqs. (3-14) through (3-17), which should be solved simultaneously for obtaining equilibrium CANDLE burnup state. We introduce iteration scheme to solve these equations. From a given flux distribution nuclide density distributions are obtained using Eq. (3-14), then with these values more consistent value of neutron flux is obtained from Eqs. (3-15) and (3-16), and this procedure is repeated until it converges. However, in usual cases, the exact value of V is unknown. Then an initial guess is introduced, and this value should be improved at each iteration stage. If the employed value of V is not correct, the shapes of neutron flux and nuclide densities are expected to move along z-axis. Therefore, the value of V can be modified from the value of distance by which those shapes move per each iteration stage.

In order to define the position of these shapes, the center of neutron flux distribution is introduced. It is defined as

$$\vec{r}_C = \frac{\int_V \phi(\vec{r})\vec{r}d\vec{r}}{\int_V \phi(\vec{r})d\vec{r}} \tag{3-37}$$

Now the way of modification of V at each iteration step is discussed. When the velocity $V^{(i)} \neq V$ is employed, the shape is considered to move with the velocity proportional to $V^{(i)} - V$ from the analogy of Eqs. (3-10) and (3-13). One cycle of iteration corresponds to passing the time proportional to $\Delta z / V^{(i)}$ considered from Eq. (3-13), where Δz is mesh width of the z-axis. When z coordinate value of $\vec{r}_C$ is obtained as $z_C^{(i)}$ for the i'th iteration for a given velocity $V^{(i)}$, the following relation can be expected:

$$\Delta z_C^{(i)} = \alpha(V^{(i)} - V) / V^{(i)} \tag{3-38}$$

where $\Delta z_C^{(i)} = z_C^{(i)} - z_C^{(i-1)}$ and α is a constant. From this relation we can derive the following equation to get a proper estimate of V for the (i+1)'th iteration using the results for the i'th and (i-1)'th iteration:

$$V^{(i+1)} = V^{(i)} V^{(i-1)} \frac{\Delta z_C^{(i)} - \Delta z_C^{(i-1)}}{\Delta z_C^{(i)} V^{(i)} - \Delta z_C^{(i-1)} V^{(i-1)}} \tag{3-39}$$

Now we have a whole iteration scheme, but it is required to use two good initial guesses of V, $V^{(1)}$ and $V^{(2)}$. They can be estimated from several trial calculations with different values of V', where the value of V' is fixed during the iteration. Here Eqs. (3-14) through (3-17) are solved repeatedly. If the height of core is infinity, the burning region moves forever for $V' \neq V$. In this calculation stage for finding two good initial guesses, large but finite height is treated, and zero-flux boundary condition is set for both upper and lower core boundaries. Then, the move of burning region stops finally after several iterative calculations even for $V' \neq V$, since the boundary condition does not permit the burning region to pass the boundary. The iteration is finally converged. If the value of tried V' is more different from V, then the burning region of core moves more until it converges and arrives nearer to the boundary. The case in which the burning region stays nearer to the boundary gives smaller value of k_{eff}, since the neutron leakage becomes larger. Therefore, V' value with which k_{eff} becomes the largest should be the best candidate of the initial guess of V among all trial values. By using the best two values of V as the initial guesses starts the iterative calculation mentioned above.

3.5. Calculation System Employed in This Book

In this book all calculations have been performed by using dedicated computer code to solve Eqs. (3-14) through (3-17) for *r-z* two dimensional geometry by using multi-group diffusion treatment for Eqs. (3-15) through (3-17) and modified Runge-Kutta method for Eq. (3-14). The method to solve time-independent coupled system of neutron diffusion and nuclide burnup are similar to the methods employed in the previous equilibrium in-core fuel management analyses [Sekimoto, Pigford, 1974; Sekimoto, et al., 1987; Obara, Sekimoto, 1991]. It can be considered exact enough for the present purpose.

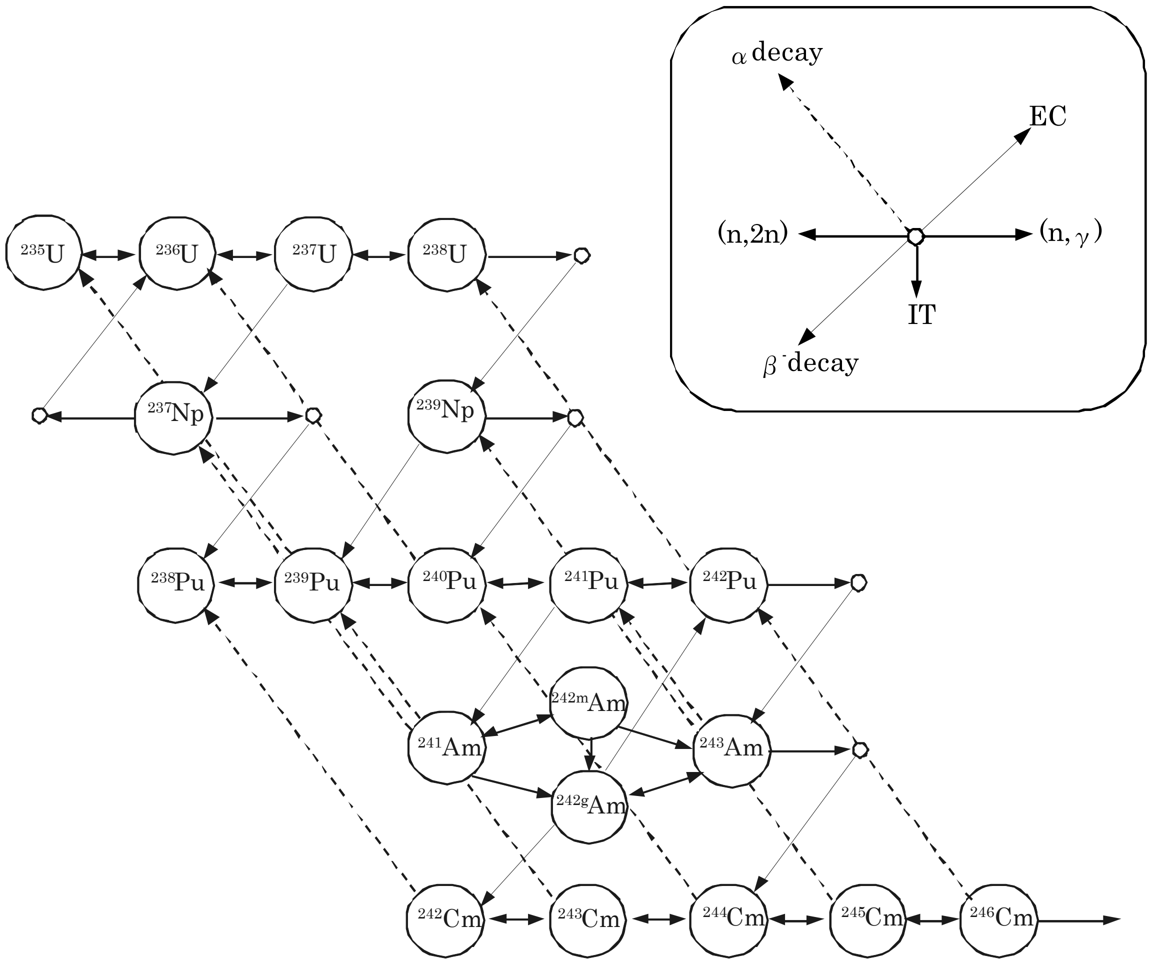


Figure 3-1. Actinide nuclides chain

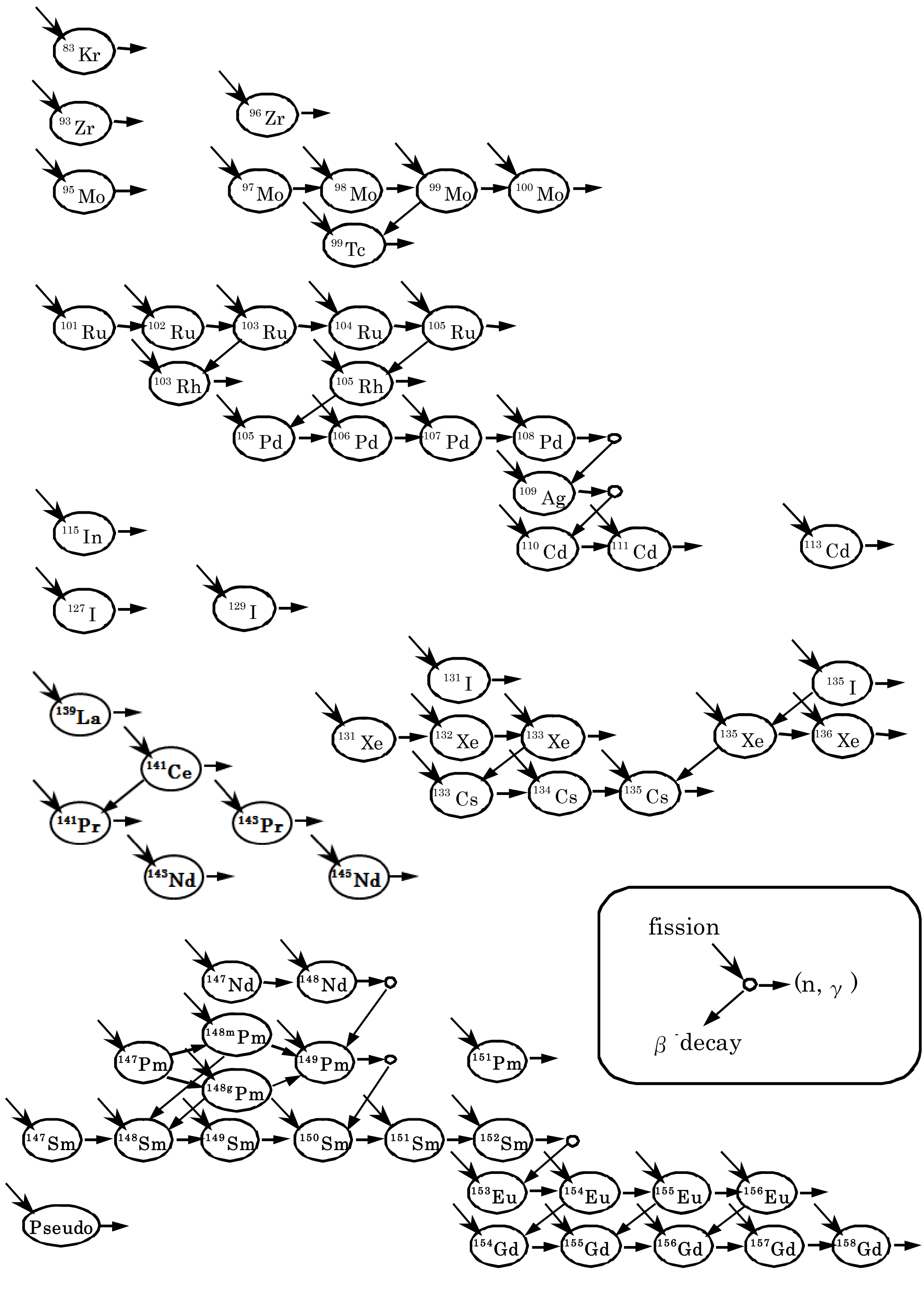


Figure 3-2.　Fission product nuclides chain

In the present calculation 20 actinides and 66 fission products are employed as shown in Figure 3-1 and 3-2, respectively, for fast CANDLE reactors. The capture cross section of the nuclides produced by neutrons capture of the nuclide at the end of the nuclide chain is assumed to be the same as the cross section of the nuclide at the end of the chain. It is equivalent to that the nuclide at the end of the chain remains the same nuclide even after capturing neutrons.

The group constants and their changes with respect to temperature and atomic density are calculated using a part of SRAC code system [Okumura, et al., 1996] with JENDL-3.2 nuclear data library [Nakagawa, et al., 1995]. In the CANDLE reactor the microscopic group cross-sections are changed for different spatial positions in the core caused by the changes of the fuel composition and temperature. In our previous study [Sekimoto, Udagawa, 2006] the new method is developed, where the microscopic group cross-sections are evaluated at every space mesh by table look-up and linear interpolation method, and used to analyze a fast CANDLE reactor with natural uranium as a fresh fuel. The results are compared with the conventional method, where only one set of the microscopic group cross-sections is employed, to investigate the effects of space-dependency of the microscopic group cross-sections and feasibility of the old method. The differences of the effective neutron multiplication factor, burning region moving speed, spent fuel burnup and spatial distributions of nuclide densities, neutron flux and power density may be considerable from the reactor designer point. However, they are small enough when we study only about the characteristics of CANDLE burnup for different designs. In most cases we will use the simple code system.

4. Parametric Studies of CANDLE Reactors in Equilibrium State

Both fast and thermal CANDLE reactors were considered in Chapter 2. However, Chapters 4 to 9 focus only on fast CANDLE reactors since they exhibit much more attractive features than thermal CANDLE reactors. Even when discussing fast CANDLE reactors, there are many different designs that can be considered; for example, several kinds of fuels and moderators can be used and the core size and cell designs can be varied. This chapter shows how the performance of a CANDLE reactor varies when these parameters are varied. The performances are compared in the equilibrium state.

4.1. Reference Design

This chapter considers parametric studies of CANDLE burning for fast CANDLE reactors. Table 4-1 shows the reference reactor design parameters.

Table 4-1. Design Parameters of Reference Reactor Design

reactor	thermal output	3000MWt
	core radius	200cm
	radial reflector thickness	50cm
fuel pin	fuel form	U-10w%Zr
	fuel pellet diameter	0.8 cm
	cladding tube material	HT-9
	cladding tube thickness	0.035cm
coolant		Pb-Bi (44.5%,55.5%)
fuel volume fraction		50%

Its total thermal power output is 3 GW. The core is cylindrical with a radius of 2 m and an infinite height (since an infinite height is considered as an unambiguous standard, as mentioned in Chapter 2). However, setting the core height to infinity is not possible in practical calculations, so instead it was set to 8 m. When the core height used in the calculation is sufficiently large that the neutron flux and leakage are negligibly small at the top and bottom core boundaries, the neutron flux distribution in the burning region will be unaffected by a change in the core boundaries and all the distributions can be considered as those for an infinitely long core. In our study it was confirmed by investigating the effects on the solution of changing the boundary conditions from vacuum to reflective conditions. The actual core height is discussed in relation to the economical design in subsequent chapters.

Natural uranium was used as the fresh fuel. The use of natural or depleted uranium as the fresh fuel has a negligible effect on the reactor performances, since ^{235}U has a much smaller contribution than the nuclides produced from ^{238}U, as Figure 2-5 shows. The fuel is metallic and contains 10% Zr.

Since CANDLE burning requires an excellent neutron economy, a lead-bismuth-eutectic (LBE) cooled metallic fueled reactor was used as the reference reactor as it has a hard neutron spectrum that results in a high neutron economy. For the same reason, the percentage of fuel volume was set to 50%, which is larger than that in current reactors; this reduces the cooling capacity of the coolant.

4.2. Parametric Surveys

This section compares calculation results for important design parameters that differ from those of the reference design described in the previous section. The fuel and coolant materials, fuel volume fraction, and core radius are varied, and the effective neutron multiplication factor, k_{eff}, the burning region moving speed, V, and the average burnup of spent fuel are calculated and compared. Table 4-2 shows the obtained results. The value of V is very low, being about 4 cm/year in all cases. The average burnup of spent fuel was extremely high, being about 400 GWd/t, which indicates that about 40% of the loaded fuel was burnt. These results are expected based on the discussion in Section 3.3. The values are similar in all cases.

Table 4-2(a) shows that only the k_{eff} of the metallic fuel exceeds unity. It is clearly difficult to realize CANDLE burning using oxide fuel, whereas it may be possible with a little effort using nitride fuel. Both V and the average burnup of spent fuel differ for different fuels, but V varies more. This large variation in V is attributed to the different fissile material densities of the fuels.

Table 4-2. Effective neutron multiplication factor, burning region moving speed, and spent fuel burnup for various core design parameters.

(a) For different fuels

Fuel	Oxide	Nitride	Metal
Effective neutron multiplication factor	0.926	0.990	1.015
Moving speed of burning region	4.7 cm/year	3.5 cm/year	3.8 cm/year
Average burnup of spent fuel	452 GWd/t	445 GWd/t	426 GWd/t

(b) For different coolants

Coolant material	Sodium	Lead	Lead bismuth	Helium
Effective neutron multiplication factor	1.006	1.012	1.015	1.035
Moving speed of burning region	3.8 cm/year	4.1 cm/year	3.8 cm/year	3.8 cm/year
Average burnup of spent fuel	415 GWd/t	427 GWd/t	426 GWd/t	413 GWd/t

(c) For different fuel volume fractions

Fuel volume fraction	40%	50%	60%
Effective neutron multiplication factor	0.989	1.015	1.035
Moving speed of burning region	4.8 cm/year	3.8 cm/year	3.2 cm/year
Average burnup of spent fuel	427 GWd/t	426 GWd/t	427 GWd/t

(d) For different core radii

Core radius	150 cm	200 cm	250 cm
Effective neutron multiplication factor	0.999	1.015	1.023
Moving speed of burning region	3.8 cm/year	3.8 cm/year	3.8 cm/year
Average burnup of spent fuel	429 GWd/t	426 GWd/t	426 GWd/t

Table 4-2(b) shows that k_{eff} also varies when different coolants are used. The differences are mostly attributed to the different neutron economies of the coolants. Helium has the highest k_{eff} and sodium has the lowest k_{eff}. However, the differences are not significant and CANDLE burning is possible for any coolant when a metallic fuel is used. The cooling performance of sodium is better than the other coolants, and it will be often employed in the following parts of this book, though the CANDLE burnup can be realized more easily than the others.

The effect of fuel volume fraction is shown in Table 4-2(c). When the fuel volume fraction is large, k_{eff} is large so that CANDLE burning can be realized more easily. The change in V is inversely proportional to the fuel volume fraction, but the average burnup of spent fuel does not change for different fuel volume fractions.

Table 4-2(d) shows that the core radius also exerts a considerable influence. When the core radius is large, k_{eff} is large so that CANDLE burning can be realized more easily. However, neither V nor the average burnup of spent fuel change for different core radii.

4.3. Advantages and Problems

Based on the above results, the advantages of using CANDLE burning in a fast reactor with an excellent neutron economy are summarized. The results reveal the following advantages, which defy the common wisdom regarding conventional nuclear reactors.

1) It is possible to design a reactor that does not require fissile fuel except in the initial core.

   Accordingly, natural or depleted uranium suffices as fuel for all the cores after the initial core. Thus, if there is sufficient fissile material for the initial core, no enrichment or reprocessing facilities are required. Furthermore, no waste will be generated by these facilities.

2) The average burnup of spent fuel in this reactor is about 40%.
   - 40% of natural (or depleted) uranium fissions generate energy without enrichment and reprocessing. It will be discussed again in Section 5.2.
   - This value corresponds to that of a currently planned, conventional fast reactor/reprocessing system (with 70% utilization of natural uranium).

- Even for a simple once-through cycle, fuel resources will be 60 times greater and the waste for geological disposal will be 10% that of presently used LWRs (with a burnup rate of 4% for 4% enriched fuel, corresponding to 0.7% of the original natural uranium). Miscellaneous waste, especially which associated with fuel reprocessing, will also be extremely small. It will be discussed again in Section 5.2.
- It is important to explain in some detail the use of depleted uranium presently stored at enrichment facilities. Until now, enriched uranium fuel has been prepared for use in LWRs. Consequently, there is currently a large amount of depleted uranium in storage. 82% of the original natural uranium is converted into depleted uranium. If 40% of this can be burnt, then 33% of the original natural uranium can be utilized by CANDLE burnup. The use of enriched uranium (18% of the original natural uranium) in a LWR with 4% burnup means that 0.7% of the original natural uranium is used. In other words, using depleted uranium in a CANDLE reactor can generate 45 times more energy than has been produced until now. It will be discussed again in Section 5.2.

3) The speed of the movement of the burning region, V, with burning is about 4 cm/year, which makes it easy to design a long-life reactor.

To increase the core life by 20 or 30 years, the core height just needs to be increased by 0.8 or 1.2 m. The value of V changes for different designs as mentioned in Section 4.2 and takes very low value for lead or lead-bismuth cooled reactors as shown in Section 6.3 making easier long-life reactor.

4) Even when a core disruptive accident occurs, it is less likely to become a recriticality accident.

There is no need for a neutron absorber or reflector to control the excess reactivity. There is also no excess fissile material in the core to produce excess reactivity. The amount of coolant, which may suppress k_{eff}, is very small. Therefore, even if the core is disrupted and fuel rearrangement occurs, it is less likely to lead to a recriticality accident. It will also be discussed again in Section

5.2.

However, there are the following potential problems:

1) A reactor design with an excellent neutron economy is necessary.
   This is a significant problem. However, we have already introduced some design examples that are considered to be currently achievable. It is important to consider ways of controlling the reactor, including power level regulation and shut down. The equipment required for these operations may reduce the neutron economy, thereby making CANDLE burning difficult. It is generally difficult to design a fast reactor that has an excellent neutron economy with a negative power reactivity coefficient. It is hence important that this problem be investigated further.
   The author has several ideas to solve these problems. However, it is not the subject of this book and they are omitted from this book.

2) It is necessary to ensure material integrity under more than 40% burnup.
   It might be necessary to use materials that can withstand over 50% burnup. There is currently no fuel element material that can withstand such a high burnup. This problem is discussed in Chapter 8. The volume of accumulated fission products becomes large with high burnup. In this state, the pressure in the fuel element will become excessively high, making it necessary to release gas from there. This necessitates a major design change. Even if the gaseous fission products can be managed, the solid fission product volume will also become high, especially when metallic fuel is used; to counteract this, the fresh fuel density will need to be reduced.
   Considerable material development research is required. However, even though present claddings can withstand burnups of much lower than 40%, CANDLE burnup can be realized by employing simple reprocessing such as the DUPIC fuel-handling technique [Choi et al., 2001], as shown in Figure 4-1. DUPIC is a dry process that does not separate actinides and fission products. In addition,

volatile fission products are released from the fuel and the cladding is replaced by a new one.

This problem will be discussed again in Section 8.1.

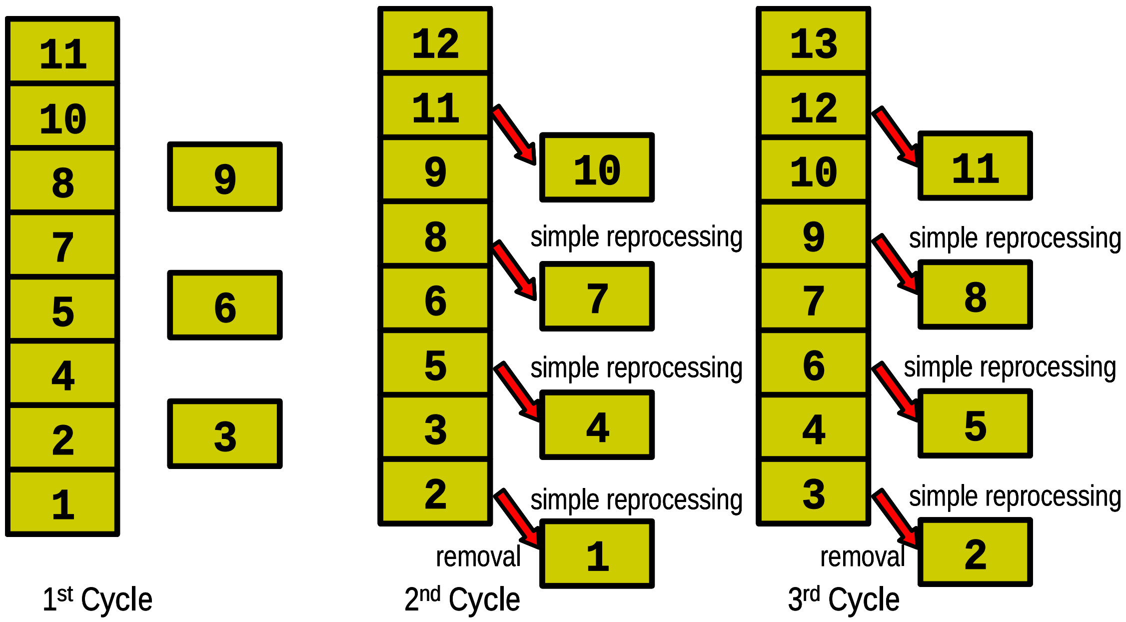


Figure 4-1. An example of a CANDLE fuel cycle: In the first cycle, fuel element 1 is fresh fuel and fuel elements 3, 6, and 9 undergo simple reprocessing. In the second cycle, fuel element 1 is removed from the core and fuel element 2 is moved down to its place. Fuel elements 3, 6, and 9 are moved to the positions previously occupied by fuel elements 2, 5, and 8, respectively. Fuel element 12 is charged to the core. In the third cycle, fuel element 2 is removed and similar refueling is repeated.

5. Future Ideal Nuclear Energy System

5.1. Requirements for Nuclear Reactors

There are dwindling supplies of concentrated energy sources such as fossil fuels. Nuclear energy sources are highly concentrated energy sources and they also are in short supply, if only LWRs are used. Nuclear reactors produce a lot of radioactive materials through nuclear reactions. They cause the problem of accidents during reactor operation and the problem of radioactive waste disposal after reactor operation. Another inherent problem of nuclear energy generation is that it employs materials and technologies that are closely related to atomic bomb production. Problems associated with atomic bombs include the need to implement safeguards, terrorist threats, and nuclear proliferation. Cost effectiveness is an important requirement for energy. Thus, for nuclear energy to be utilized as a primary energy all the problems associated with a) limited resources, b) safety, c) waste disposal, d) bomb manufacture, and e) cost need to be solved (see Figure 5-1) [Sekimoto, 2009a,b, 2010].

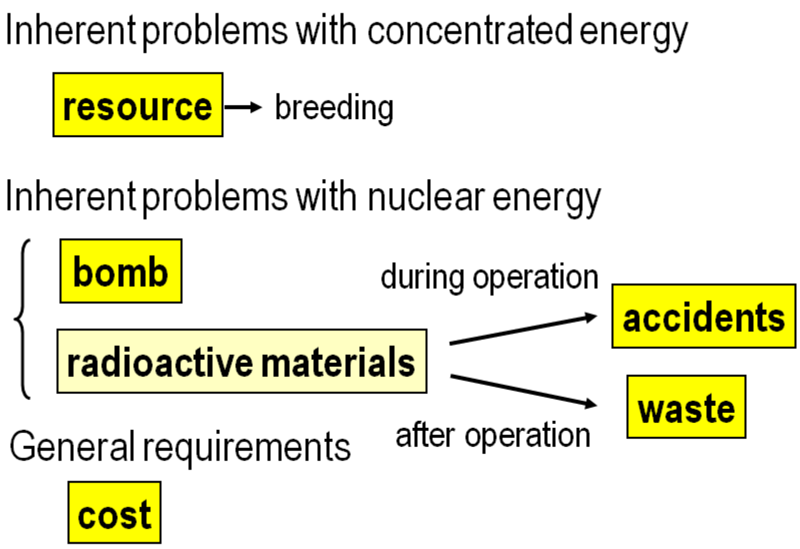


Figure 5-1. Necessary and sufficient requirements for nuclear energy systems.

5.2. How CANDLE Reactors Satisfy these Requirements

A very high neutron economy is required to realize CANDLE burning in fast reactors. In our previous studies, we found that only very hard neutron spectrum fast reactors can realize this burning. However, once it is realized, natural or depleted uranium can be used as replacement fuels and 40% of it can be burned up without reprocessing.

As shown below, CANDLE reactors can overcome all the problems mentioned in the previous section regarding a) limited resources, b) safety, c) waste disposal and d) bomb manufacture, [Sekimoto, 2009a,b, 2010; Sekimoto, Nagata, 2010]:

a) Resource

The burnup of spent fuel is about 40% (400 MWd/tHM); in other words, 40% of natural uranium burns up without enrichment or reprocessing. This value is competitive with that of presently planned fast reactor systems with reprocessing plants.

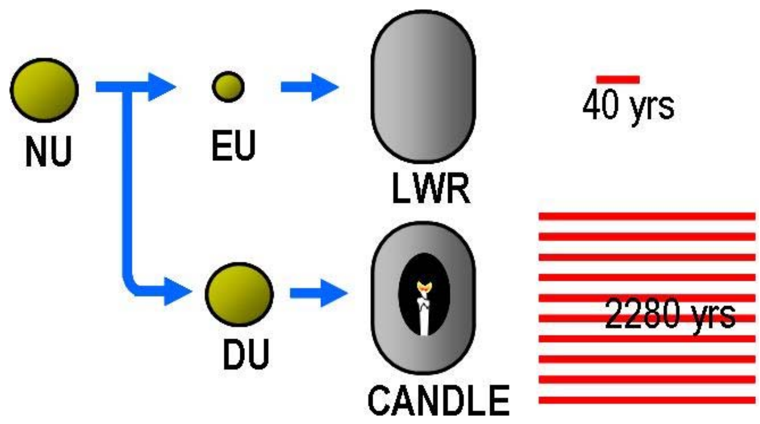


Figure 5-2. CANDLE reactor operation after LWR operation.
NU: natural uranium, EU: enriched uranium, DU: depleted uranium

The present once-through fuel cycle of 4% enriched uranium in LWRs burns up about 4% of the inserted fuel, which corresponds to a utilization rate of about 0.7% of natural uranium (this depends slightly on the enrichment of depleted uranium). In this case, about 87% of the original natural uranium remains as depleted uranium. If this depleted uranium is utilized as fuel for a CANDLE reactor, 35% (=0.87×0.4) of the original natural uranium will be utilized. Therefore, if an LWR has already generated x Joules of energy, a CANDLE reactor can produce about $50x$ Joules from the depleted uranium stored at an enrichment facility for LWR fuel.

Based on the discussion in Section 4.3, the following scenario is conceivable. If LWRs produce energy sufficient for 40 years and the nuclear energy production rate does not change in the future, we can produce energy for more than 2000 years by using CANDLE reactors (see Figure 5-2). Thus, it is not necessary to mine any more uranium ore and reprocessing facilities are not required.

b) Safety

Most discussions of safety are controversial, making it difficult to reach constructive conclusions. This is because many different factors affect safety and safety criteria are often not clear. This section seeks to discuss safety issues clearly. Since it is impossible to guarantee absolute safety in practical situations, we discuss relative safety and attempt to demonstrate that our reactor is safer than conventional power reactors.

If one reactor has a lower frequency of undesirable events and less serious consequences of the most severe accident than another reactor, the first reactor can be said to be safer than the second one. This section discusses the frequency of undesirable events and the consequences of the most severe accident of CANDLE reactors:

1) Frequency of undesirable events

   Most accidents in nuclear facilities such as nuclear reactors are caused by human errors. Complicated systems generally induce human errors. As shown below, the CANDLE reactor is a very simple reactor and it does not impose any difficult demands on operators.

   Firstly, CANDLE burning does not require any burnup reactivity control mechanism. This makes reactor control simple. Its excess burnup reactivity will be zero and there will be no reactivity-induced accidents under normal operating conditions.

Secondly, the number density distribution of each nuclide does not vary with burning in the burning region. Therefore, the reactor characteristics such as power peaking and the power coefficient of reactivity do not change with burning. It is thus possible to accurately estimate core conditions. The reactor operation strategy is the same in each burning stage.

Thirdly, since the radial power profile does not change with burning, the required flow rate for each coolant channel does not change. Therefore, orifice control with the progression of burning is not required. Thus, operational errors are avoided.

Furthermore, depleted or natural uranium is used as the fresh fuel after the second cycle. This makes transportation and storage of fresh fuels easy in relation to criticality and physical protection. Uranium mining is also dangerous for mine workers; CANDLE burning can use depleted uranium instead of natural uranium obtained from a mine.

2) Most severe accident

Recriticality accidents occurring after core disruptive accidents are considered to be the most severe accidents of fast reactors. CANDLE burning considerably reduces the possibility and consequences of recriticality accidents after the core disruptive accidents since control rods are not inserted in the core and there is little coolant in the core.

Thus, both the frequency of undesirable events and the consequences of the most severe accident for CANDLE reactors are smaller than those of current fast reactors, implying that CANDLE reactors are safer than conventional fast reactors.

c) Waste disposal

LWRs currently achieve a burnup of about 4% for the inserted fuel of 4% enriched uranium. In contrast, the burnup of the spent fuel of CANDLE reactors is about 40%, which is 10 times greater than that for LWR. Therefore, the amount of spent fuel per energy generated is about one tenth that of a once-through cycle LWR.

Separation of high-level waste components from spent fuels and vitrifying them may reduce the volume of high-level waste, but it increases the total volume of radioactive waste. These processes generate a lot of waste that is contaminated with radioactive materials. The once-through fuel cycle of a CANDLE reactor reduces the total volume of radioactive waste.

The amount of actinides is reduced since they are stored in the core much longer than for conventional reactors, during which time they undergo a considerable amount of fission.

Furthermore, since a CANDLE reactor can use depleted uranium, no waste is generated by uranium mining.

d) Bomb manufacture

Enrichment and reprocessing are the two most important technologies for manufacturing bombs. After they have been started, CANDLE reactors can be operated indefinitely without enrichment or reprocessing if only natural or depleted uranium is available. Therefore, CANDLE reactors are suitable for overcoming problems such as nuclear safeguards, terrorist threats, and proliferation of nuclear material.

e) Cost

Nuclear reactors have costs associated with capital, fuel, and operation and maintenance. CANDLE reactors are expected to have low operation and maintenance costs since they have simple designs. They are also anticipated to have low fuel cycle costs, since it is not necessary to reprocess discharged fuel. The capital costs are also expected to be lower, but not by a large amount since the core is the only major difference from conventional reactors.

The costs are highly dependent on the power rate. This dependence becomes very large when interest rates are high. In this case, the power rate is equivalent to the power density. Therefore, we compare the power density with those of conventional fast breeder reactors such as Super Phoenix and Monju. Since CANDLE reactors have lower coolant channel volume ratios than conventional reactors, they have poorer core cooling performances. This may result in a lower average power density. On the other hand, CANDLE reactors do not have blankets, and also their radial power shapes are not distorted by control rods. Therefore, they have very smooth radial power distributions. The design optimization shown in Chapter 9 can be realized with a short core height and a radially flat power distribution. The latter effect (better power density distribution) exceeds the former effect (poor cooling performance), and the fuel charged region of CANDLE reactors has much higher average power densities than current fast breeder reactors.

Based on the above discussion, we conclude that the CANDLE reactor can supply cheaper energy than current fast breeder reactors.

The cost depends on the technologies available for realizing ideal designs. For CANDLE reactors, a material that can withstand high burnups is a critical issue. This is discussed in Chapter 8.

6. Small Long-Life CANDLE Reactors

6.1. Small Long-Life Reactors

Conventional nuclear power reactors have almost reached the upper limit in terms of minimizing their costs by increasing their scale. It may be difficult to find ways to this direction to improve their performance. It will soon become almost impossible to find suitable sites for them in developed countries. They also entail high economic risks that are too large even for large companies and governments to bear. It is much easier to find suitable sites for smaller reactors since they can be constructed on lower grade land that is smaller and less stable than that required by larger reactors.

Small reactors can also be utilized for various purposes besides electricity generation, including heat generation and desalination. Transporting heat and pure water over long distances is expensive and involves energy and material losses. Thus, small reactors are better suited as local reactors for such purposes since they have low power requirements.

The small scale of small reactors considerably reduces their economic performance. However, there are many factors pertaining to small reactors that can enhance their economic performance. Some small reactors can be constructed in factories, which considerably reduce reactor costs. For a given power rate, more small reactors are required than large reactors. Therefore, it is possible to obtain more knowledge about small reactors based on more experience. Smaller reactors have shorter licensing and construction periods and smaller interest on investments. Modular systems are expected to be efficient and have excellent economic performances.

The long life of small reactors has many advantages, but it can also be an economic disadvantage since there will be considerable interest on fuel costs. However, long-life reactors also have many economic advantages. For example, long-life reactors do not require expensive refueling systems that are generally installed in reactors.

Maintenance costs will also be lower. In addition, the high burnup reduces the fuel cycle cost and the plant operation factor will be higher.

Smaller reactors are safer than larger reactors since they have simpler systems and contain less radioactive material. Furthermore, small reactors are generally more inherently safe since their safety functions depend more on natural phenomena and less on human intervention or mechanical systems.

If a reactor is small enough to be transportable and has a sufficiently long life, it can be constructed in a factory and transported to a site. Sealing such reactors to prevent fuel discharge outside the factory will also prevent stealing of nuclear material.

In the 21st century, global warming caused by carbon dioxide emission is a serious problem. Carbon dioxide emissions from developing countries are particularly important. Nuclear reactors generate very low levels of greenhouse gases. However, developing countries have insufficient infrastructure and technicians to construct and operate nuclear reactors. Furthermore, some developing countries generally have small and local energy demands. As mentioned above, small reactors are simple to operate and maintain, are inherently safe, and are resistant to proliferation of nuclear materials.

An ideal reactor should have a long life and be safe, simple to maintain and operate, small, transportable, and resistant to proliferation of nuclear materials (i.e., it should have a sealed core). Some of these characteristics are closely related to each other. For example, transportability requires the reactor to be small; thus, transportability is considered to be a similar characteristic to smallness. The two basic characteristics are long life and smallness; all the other characteristics can be derived from these two. Therefore, in this chapter we design a small long-life CANDLE reactor.

This reactor is designed to be constructed in a factory in a developed country and then shipped to a site in a developing country where it is installed and operated for a certain period without refueling. When its operational life has finished, it can be replaced by a new reactor. An alternative is to install a reactor on a barge that can then be transported to a suitable port and operated as a power plant.

Such a small long-life CANDLE reactor requires an excellent neutron economy. Based on the results in Table 4-2(a), metallic fuel is the most suitable fuel for this reactor. Table 4-2(b) compares different coolants; however, the results may not be directly applicable to the CANDLE reactor since they are for a large reactor.

6.2. Coolant Characteristics

6.2.1. Lead and Lead-Bismuth

Sodium is widely considered to be the best coolant for fast reactors because of its superior cooling ability (see Table 6-1) and compatibility with fuel claddings. It can increase the power density and reduce the doubling time. A short doubling time was essential in the early stages of development and construction of fast breeder reactors from the 1960s to the 1980s.

Table 6-1. Characteristics of fast reactor coolants

Coolant	Melting point (at 1 atm.) (K)	Boiling point (at 1 atm.) (K)	Density at 500K (g/cm^3)	Prandtl Number
Na	371	1156	0.90	0.007
Pb-Bi	398	1943	10.45	0.029
Pb	600	2013	10.60	0.026

As mentioned previously, a good neutron economy is very important for realizing a small long-life reactor. For small fast reactors, heavy isotopes such as lead and lead-bismuth-eutectic (LBE) are expected to give much better neutron economies than sodium because of their large scattering cross sections and atomic masses. Lead or LBE cooled small long-life fast reactors have superior neutron economies, burnup reactivity swings, and void coefficients to sodium-cooled reactors [Zaki, Sekimoto, 1992]. LBE is also effective in shielding neutrons and gamma-rays, which enables the reactor size to be reduced.

The most important advantage of lead and LBE over sodium is their chemical inertness. Neither lead nor LBE react violently with water or air. Sodium boils at 1156 K (see Table 6-1), so it is difficult to prevent it boiling when severe accidents occur. If the void coefficient is positive, an accident may lead to a core destructive accident occurring. Since lead and LBE have much higher boiling points, it is very unlikely that they will boil. Furthermore, their void coefficients are more negative than that of sodium.

It is also important to consider the radioactive materials produced in the coolant during operation. For sodium, ^{24}Na should be considered. It has a half-life of 15 h and emits high-energy gamma-rays (2.8 and 1.4 MeV). Therefore, the primary loop of a sodium-cooled reactor has a very high dose rate. On the other hand, LBE does not produce so many gamma-ray emitters, although it produces polonium, which is an alpha emitter. The dose rate in the vicinity of the primary loop containing LBE is expected much lower than that for sodium.

However, it has been considered for long time in the western countries that lead and LBE cannot be used as reactor coolants based on experimental results that suggest that corrosion will occur. This problem has been solved in Russia by controlling the oxygen concentration and using LBE as a submarine reactor coolant. Eight nuclear submarines with LBE coolant have been constructed and operated for about 80 reactor years [Gromov, B., et al., 1997]. After the Russian research results were openly published, many investigations (particularly corrosion experiments) were conducted worldwide. The problem with corrosion can be overcome by a suitable choice of coolant and construction materials, temperature, flow rate, and oxygen concentration. However, the coolant velocity is currently set at less than 2 m/s. This is much less than that usually employed for sodium (about 10 m/s) and it reduces the core average power density.

Lead and LBE are about 12 times denser than sodium and their Prandtl numbers are about three times greater than that of sodium. These characteristics result in lead and LBE having low cooling capacities. From them together with the low flow velocities, lead and LBE-cooled reactors are expected to have lower power densities than sodium-cooled reactors.

The total reactor power is generally proportional to the core size, whereas the power density is restricted by some material constraints and so does not vary with core size. However, the core size cannot be reduced to less than the minimum size determined by criticality conditions. Therefore, if the total power is very low, the core size will be determined by the criticality condition and the power density will be low

[Sekimoto, 1992]. Thus, the low cooling capacities of lead and LBE are not such a serious problem for such small power reactors. Even natural circulation may be feasible for small reactors. Lead- and LBE-cooled reactors may be more suited to natural circulation than sodium-cooled reactors [Buongiorno, J., et al., 1999].

6.2.2. ^{208}Pb Coolant

Coolants with low neutron slowing-down and absorbing powers are suitable for fast reactors. A large scattering cross section is desirable for small reactors since it results in effective neutron confinement. Since lead and LBE are better in terms of these parameters than sodium, they have been used as coolants for small long-life fast reactors [Sekimoto, Zaki, 1995].

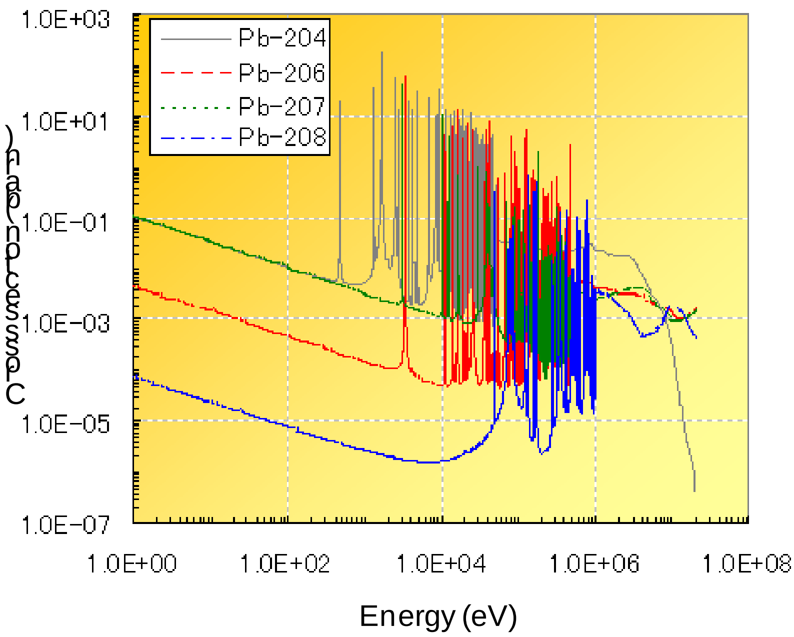


Figure 6-1. Microscopic neutron capture cross sections of four lead isotopes [Shibata et al., 2002].

Previous lead- or LBE-cooled small long-life reactors employed natural lead for economic reasons, but natural lead contains several isotopes. ^{208}Pb is a double magic nucleus and has a much smaller capture cross section (Figure 6-1) and a higher threshold energy of inelastic scattering (Figure 6-2) than the other isotopes. A ^{208}Pb-cooled metallic fuel fast reactor is expected to have a very hard neutron spectrum. The void reactivity coefficient becomes much more negative when ^{208}Pb is used as a coolant since the spectrum hardening effect by voiding ^{208}Pb becomes very small. ^{208}Pb is very expensive (\$200 kg^{-1} [Khorasanov et al., 2009]), but expensive small long-life reactors may be acceptable for special applications.

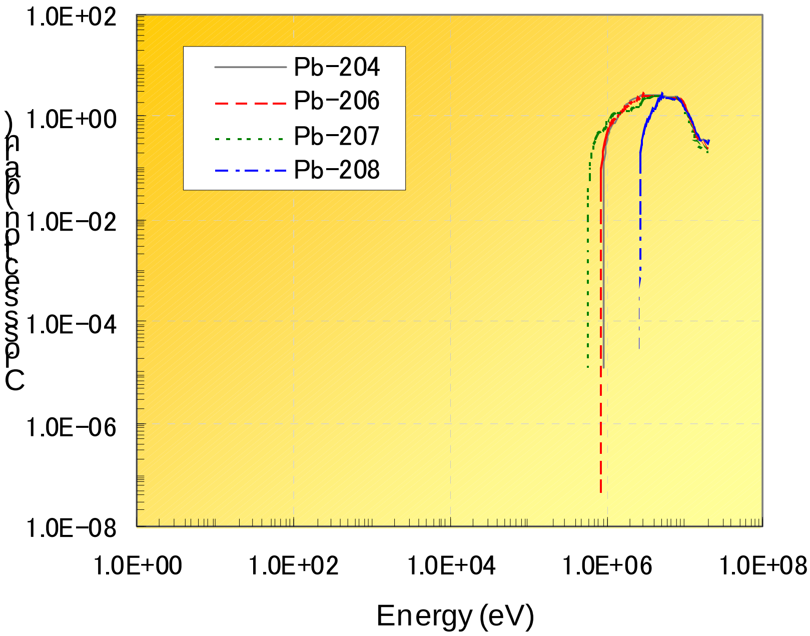


Figure 6-2. Microscopic neutron inelastic cross sections of four lead isotopes [Shibata et al., 2002].

6.3. Small Long-Life CANDLE Reactors Using Metallic Fuel and ^{208}Pb Coolant

A CANDLE reactor can be designed to have a long life since it has a very low burning region velocity (see Section 4.2). However, a CANDLE reactor requires more neutrons than conventional fast reactors since it does not employ reprocessing. It is thus important to use neutronically superior materials. This chapter presents an example of a small long-life CANDLE reactor that uses metallic fuel and ^{208}Pb coolant. It also discusses the feasibility of natural coolant circulation.

An integral reactor design is employed in which steam generators are installed in the reactor vessel since the lead core coolant is not expected to react violently with the water coolant for the steam generator. Mechanical centrifuge pumps are also inserted in the reactor vessel to generate forced circulation. Table 6-2 shows the design parameters and calculation results for the reactor.

Table 6-2. Design parameters and calculation results for a small long-life CANDLE reactor

Thermal power rate	440 MWt
Electric power rate	179 MWe
Fuel	U–10wt% Zr
Smear density	85%TD
Cladding	HT-9
Cladding outer diameter/thickness	1.50 cm/0.5 mm
Bonding material	Sodium
Pin array	Triangular
Pitch to diameter ratio (P/D)	1.2
Coolant	Pb-208 (95% enriched)
Maximum coolant velocity	2.0 m/s
Coolant inlet/outlet temperature	$400^{o}C/550^{o}C$
Core diameter/height	2 m/1.5 m
k_{eff}	1.001
Average power density	93 W/cm^3
Burning region velocity	2.6 cm/year

If natural LBE is used the coolant instead of ^{208}Pb, P/D should decrease to 1.14, making the reactor critical (k_{eff}=1.001). This reduces the average power density to 77 W/cm^3 and the thermal and electric power rates to 363 MWt and 148 MWe, respectively.

If natural circulation is employed instead of forced circulation, the maximum coolant velocity will be 0.63 m/s and the average power density will be 29 W/cm^3. The thermal power rate, electric power rate, and burning region velocity will then be 139 MWt, 56 MWe, and 0.82 cm/year, respectively.

7. Initiation of CANDLE Burning

7.1. Several Methods for Attaining Equilibrium State of CANDLE Burning

The above demonstrations of CANDLE burning only considered equilibrium steady states since they typically exhibit CANDLE characteristics. However, the following question may be asked: “Equilibrium states exhibit good properties, but how can they be obtained? Is it possible to assemble a suitable initial core for attaining equilibrium?” The equilibrium core contains many radioactive materials including higher actinides and fission products. It is difficult to determine the amounts of these radioactive materials when constructing the initial core. It may thus seem to be difficult to construct the initial core, but there are several methods for overcoming these problems.

The initial core may be realized by supplying a sufficient number of neutrons from an external neutron source [Teller et al., 1996]. However, this method is expensive and the power profile varies drastically in the early stages of the first cycle. By using enriched uranium [Sekimoto, Miyashita, 2005, 2006] and/or plutonium [Sekimoto et al., 2003] substituted for actinides in the equilibrium core, it may be possible to construct an initial core that has a similar power profile to the equilibrium one and that can reach an equilibrium state without any drastic changes.

The isotopic fraction of plutonium (plutonium vector) varies for different spent fuels, which makes it difficult to construct the first core for CANDLE burning using plutonium from spent fuel. On the other hand, uranium enrichment can be precisely controlled, so that it is easier to construct the first core. Below, a first core is considered in which enriched uranium is substituted for actinides in the equilibrium state. Reprocessing and enrichment plants are respectively required to produce plutonium and enriched uranium. A further advantage of using enriched uranium instead of plutonium in the first core is that it is easier to maintain and operate an enrichment plant than a reprocessing plant.

7.2. Initial Core Composed of Enriched Uranium

Table 7-1 shows the design parameters of the CANDLE reactor used to investigate the startup problem. It employs ^{15}N enriched nitride fuel and LBE coolant. Nitride fuel is popular for LBE-cooled fast reactors in Russia. However, it has a worse neutron economy than metallic fuel, as Table 4-2(a) shows. Consequently, tube-in-shell fuel [Hiraoka et al., 1991] was employed to increase the fuel volume fraction.

Table 7-1. Design Parameters of CANDLE Reactor for Studying the Startup Problem

Reactor	Thermal output	3000 MWt
	Core radius	200 cm
	Radial reflector thickness	50 cm
Fuel	Fuel type	Tube-in-shell
	Fuel material	^{15}N enriched nitride
	Coolant channel diameter	0.668 cm
	Coolant channel pitch	1.132 cm
	Cladding tube material	HT-9
	Cladding tube thickness	0.035 cm
Coolant		Pb-Bi (44.5%,55.5%)

In our design procedure for the initial core, the ^{238}U density is initially adjusted at each spatial mesh to conserve the fissile production rate and then the ^{235}U and niobium densities are adjusted at each spatial mesh to obtain an infinite neutron multiplication factor. The obtained nuclide densities slightly alter the neutron spectrum; the uranium enrichment was slightly adjusted to ensure that the effective neutron multiplication factor was equal to that for the equilibrium case. Figure 7-1 shows the finally obtained nuclide number densities of ^{235}U and ^{238}U. The maximum enrichment is about 13% at an axial position of about 350 to 400 cm, which is well below the constraint of 20%.

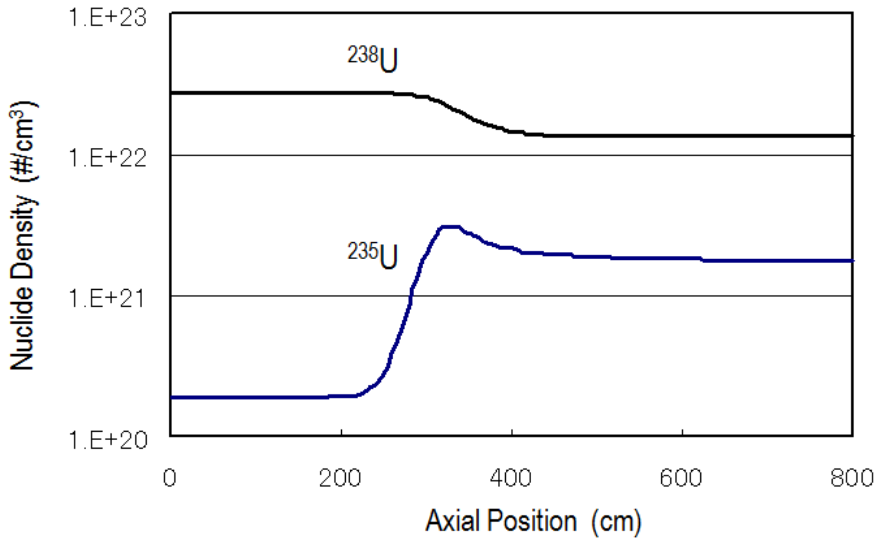


Figure 7-1. Nuclide number densities of ^{235}U and ^{238}U as functions of position on the initial core axis.

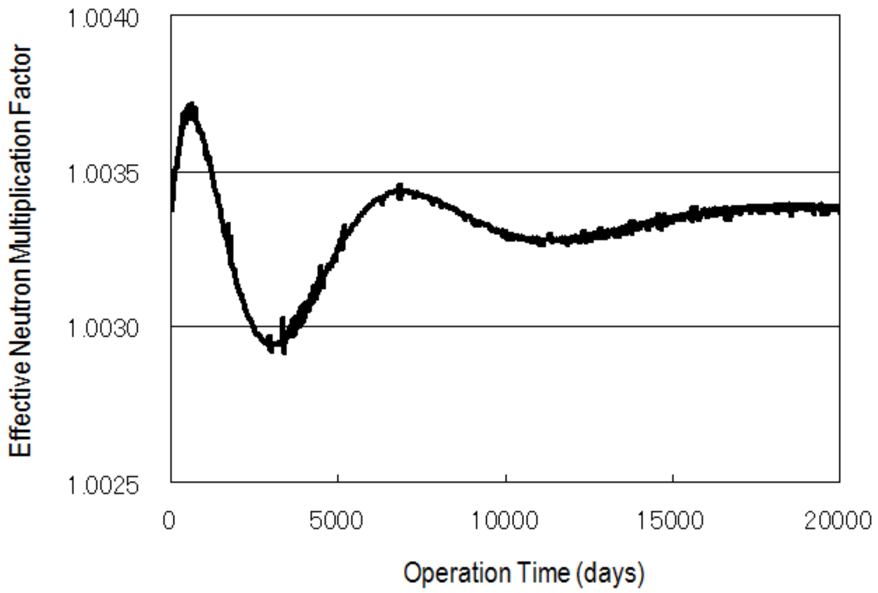


Figure 7-2. Effective neutron multiplication factor as a function of operation time.

The effective neutron multiplication factor of the obtained initial core varies with the progression of burning in the manner shown in Figure 7-2. This figure shows that the effective neutron multiplication factor oscillates with time, but that the maximum variation over the whole transient region is only 0.0008.

Figure 7-3 shows the power density distribution as a function of position on the core axis for different burnup durations. The power profile remains almost constant, but it shifts at a constant speed in the negative axial direction, as expected.

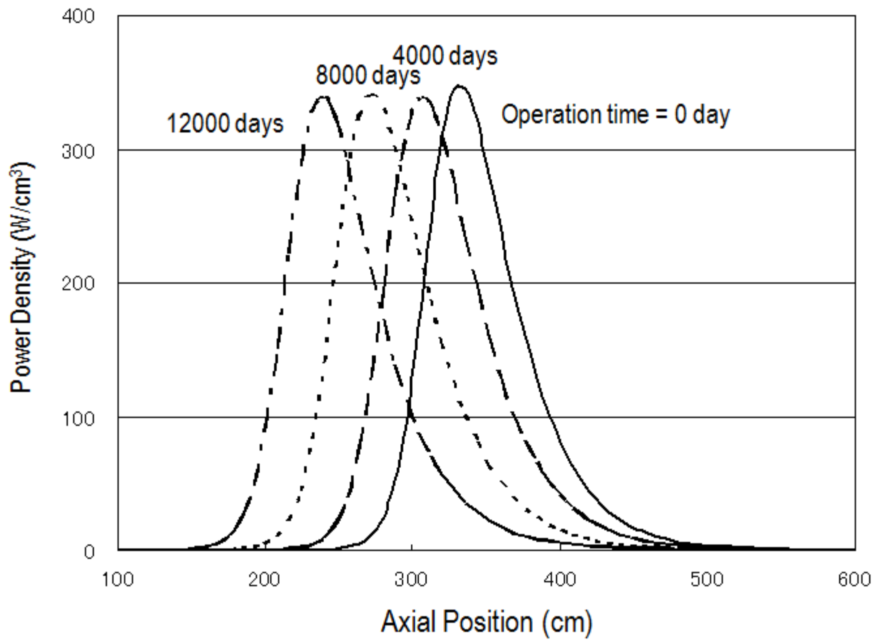


Figure 7-3. Power density distributions for four different burnup durations.

8. Problems with High Burnup

8.1. Recladding

There is currently no published data for material integrity at 40% burnup. The maximum burnup obtained in verification tests for oxide fuels used in fast reactors has steadily increased since the early 1970s. Data for 20% burnup was reported in the early 1990s. However, data suddenly stopped being published from 1994. This is probably due to the abolition of the fast reactor program in the US. For CANDLE reactors, data is required for metallic and nitride fuels; however, there is much less data available for these fuels than for oxide fuels.

The material integrity of both the fuel pellets and the cladding should be considered for high burnups. However, since we cannot expect the fuel pellets to remain intact it is necessary to rely on the cladding integrity. Even in this case, we should be concerned about pellet swelling (especially for metallic fuels) since swollen pellets may break the cladding. To reduce this problem, the fuel smear density should be reduced. To ensure the integrity of the fuel pin by means of the cladding, the displacement per atom caused by fast neutrons in the cladding material and the fission gas pressure in the pin should be considered. The displacement per atom is related to the fast neutron (>0.1 MeV) fluence; its maximum permissible value for HT-9 is about 5.0×10^{23} /cm^2, which is much less than that expected for CANDLE burnup (1.8×10^{24} /cm^2). The gas plenum volume of gaseous fission products is much larger than of conventional reactors.

As mentioned in Section 4.3, this problem could be resolved by recladding fuel elements after a certain burnup is attainted. This is depicted in Figure 8-1, where BOC and EOC denote the beginning and end of operation cycle, respectively. During recladding process volatile fission products are released from the fuel and the cladding is renewed. When recladding is utilized, the recladding frequency becomes an optional parameter. Recladding is much cheaper than reprocessing, which involves chemical processes. If the burnup for one cycle is small, recladding will be very straightforward and the separation between the cladding and the meat will be easy. However, the optimal burnup for recladding is currently unclear. The next section gives the results for an example in which the fuel element is discharged at the maximum permissible fast neutron fluence.

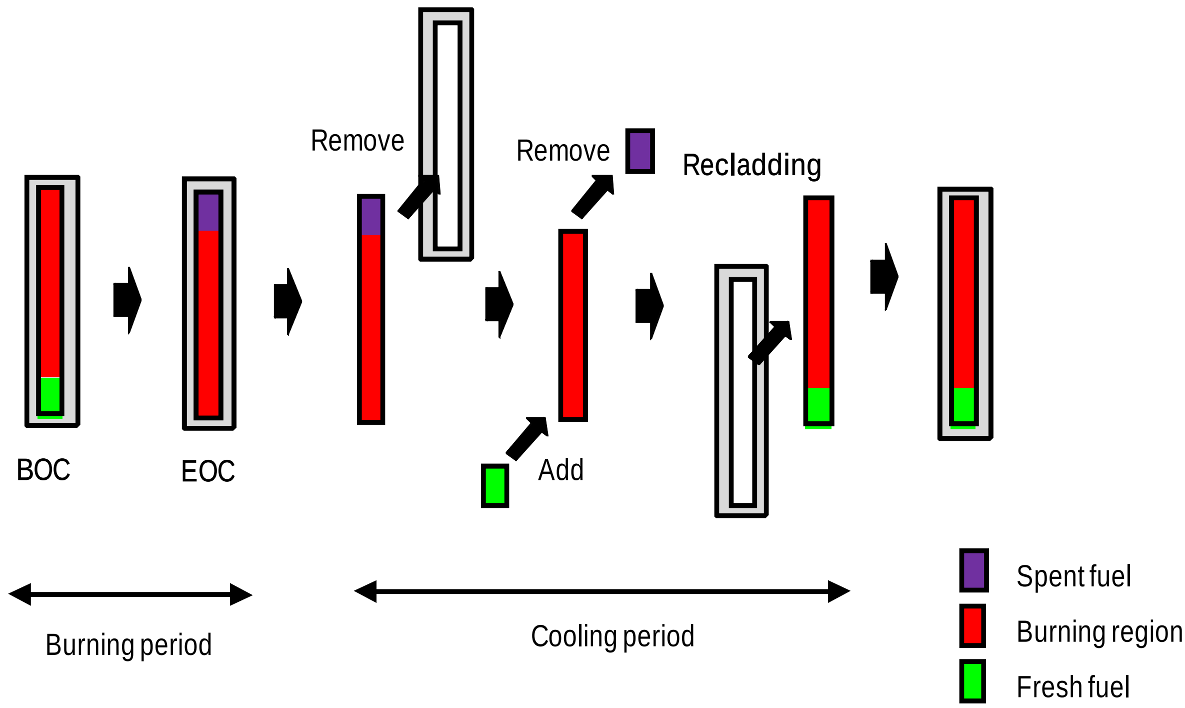


Figure 8-1. Process for recladding a fuel element (the gas plenum is not shown in this figure). At the end of operation cycle (EOC), the fuel element is discharged from the core, the cladding is removed from the fuel pin, the spent fuel region is removed, fresh fuel is added to the other end of the burning region, and the meat (i.e., the burning region plus the fresh fuel) is wrapped by new cladding. The fuel element is then charged to the core.

This chapter considers only recladding, which is anticipated to be employed in the near future. However, a more promising approach in the long term is to develop new materials that are capable of withstanding high burnups. This may result in more suitable reactor conditions being proposed.

8.2. MOTTO Cycle

To enhance the economic performance, it is important to shorten the core height, since this increases the power density and reduces the drop in the coolant pressure in the core. Figure 8-2 shows a typical steady state r–z power density distribution for CANDLE burnup in a uniform core. The axial position of the burning region shifts to up for the more distant radial position. This makes it necessary to increase the core height. To ensure that the axial power peak position is radially straight and to reduce the core height, we introduce the multichannel once through then out (MOTTO) cycle for it; detailed procedures for this method are given below:

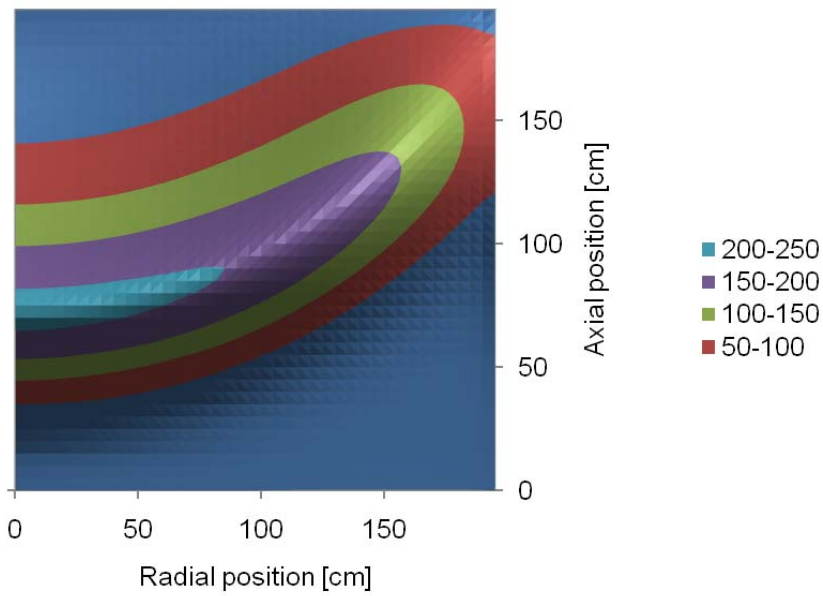


Figure 8-2. Steady-state power density distribution in a uniform infinite-height core for CANDLE burnup.

Figure 8-3 shows how the power density distribution varies in the MOTTO cycle. The centroid position is introduced for each radial position r as:

$$z_c(r) = \frac{\int_{core-bottom}^{core-top} zP(r,z)dz}{\int_{core-bottom}^{core-top} P(r,z)dz} \quad , \tag{8-1}$$

where $P(r,z)$ is the power density at the position (r,z). In Figure 8-3, H is the core height and L is the distance between the centroid position and the top surface of the core. At the beginning of power operation, the centroid line is made level by adjusting the amount of spent fuel removed at each radial position. The amount of fresh fuel added is the same as the amount of spent fuel removed at each radial position. The burning region shifts downward with the progress of burning. The speed of the downward movement is higher in the central region than at the periphery since the central region has a higher power density. Therefore, when operation is completed, the centroid line is not level: it is lower in the central region than at the periphery. This line is again made level by adjusting the amount of fuel discharged at each radial position

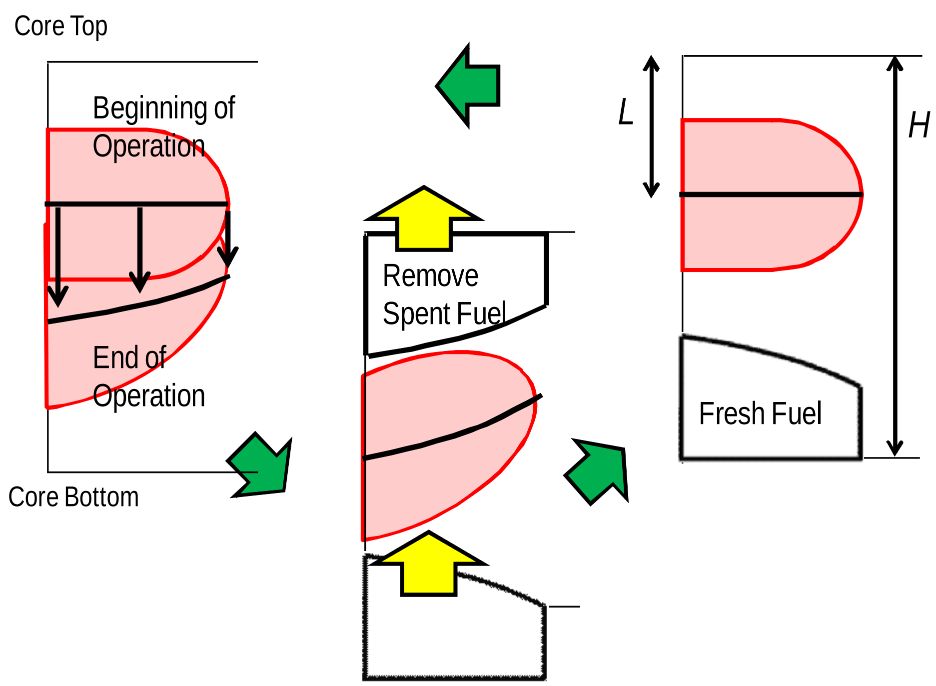


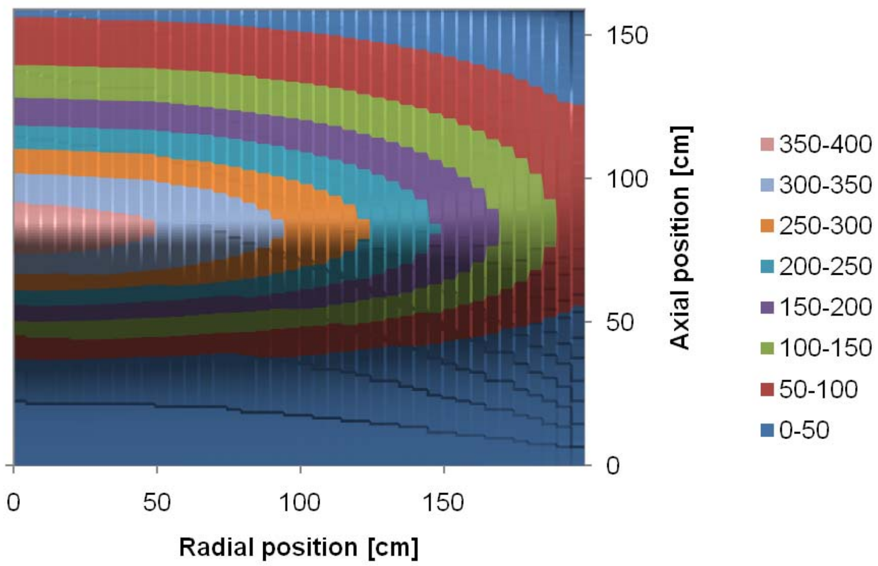
Figure 8-3 Centroid line of power density distribution at different fuel management stages.

We apply the MOTTO cycle to the reactor shown in Figure 8-2, whose core design parameters are given in Table 8-1.

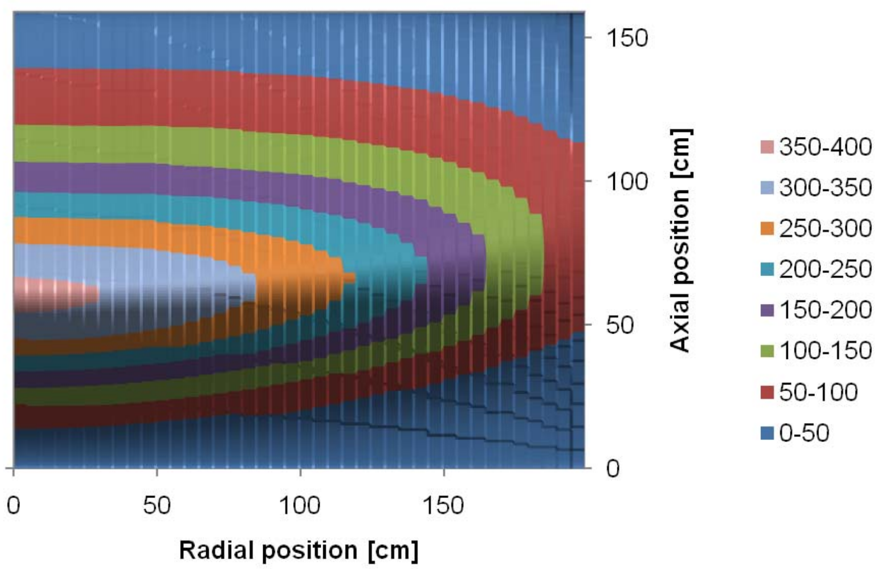
Table 8-1. Core design parameters

Reactor	Thermal output	1980 MWt
	Core radius	200 cm
	Radial reflector thickness	50 cm
Fuel pin	Fuel form	Nitride (81%TD)
	Fuel pellet diameter	1.22 cm
	Cladding tube material	ODS
	Cladding tube thickness	0.05 cm
	Pin pitch	1.45 cm
Coolant		Pb-Bi (44.5%, 55.5%)

Figure 8-4 shows the obtained power density distributions at BOC and EOC when the core height H and parameter L shown in Figure 8-3 are chosen to be 160 and 70 cm, respectively. The reactivity swing during operation is 0.0007, which is considered to be sufficiently small.



a) at BOC



b) at EOC

Figure 8-4. Power density distributions for CANDLE core (H=1.6 m, L=70 cm) using MOTTO cycle.

9. Increasing the Power Density

9.1. Power Flattening Using Thorium

A low average power density greatly reduces the economic performance of a CANDLE reactor. In this chapter, we increase the average power density by flattening the radial power profile.

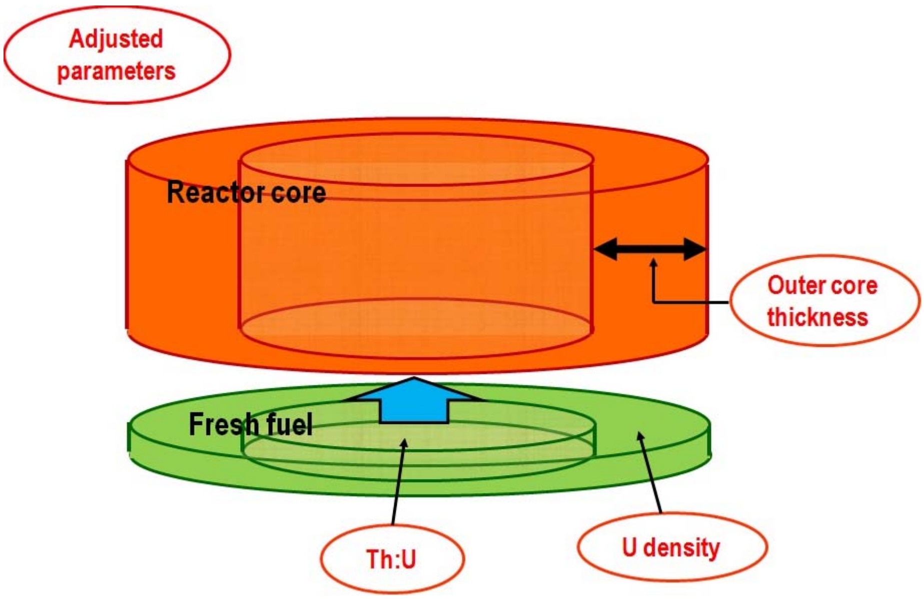


Figure 9-1. Parameters adjusted to flatten the radial power density. (The circled parameters will be adjusted.)

There are many methods that can be employed to flatten the radial power. The method used in this chapter is to uniformly add an amount of thorium to the fresh uranium fuel in the inner core, as shown in Figure 9-1. For only uranium fresh fuel, the neutron flux has a peak on the core axis and neutrons flow from the core center to the periphery. If thorium is added to the inner part of the fresh fuel region, the flux peak will decrease since the η value of ^{233}U produced from thorium is less than that of ^{239}Pu produced from ^{238}U (see Figure 1-3). By selecting an appropriate amount of thorium, the net radial neutron current in the inner core can be made to be zero. The thorium addition rates at the inner core and at the boundary between the inner and outer cores are adjusted to flatten the power density distribution in the inner core. The uranium density in the outer core can be simultaneously adjusted to make the power density distribution continuous at the boundary. Core criticality should be maintained during these processes. Neutrons leak outward in the outer region. In this way, the power density distribution in the inner core can be flattened.

9.2. Demonstrations

We treat the cases of sodium-cooled and ^{238}Pb-cooled fast reactors with metallic fuel. We evaluate only the power flattening performance of our method and we consider only reactor physics. Table 9-1 shows the design parameters of the two reactors. They have total powers of 1980 MWth and cores that are 4.0 m in diameter and 2.0 m high. The total power should be changed for different coolants in actual designs, but in this chapter we will investigate only power flattening and do not care about absolute value of total power. A 50-cm-thick stainless-steel radial reflector is introduced to the sodium-cooled reactor; the lead coolant outside the core functions as a reflector in the ^{238}Pb-cooled reactor. The other parameters were chosen to have the same values for both reactors as shown in Table 9-1, although they may be varied in the future to obtain more practical designs.

Table 9-1. Core design parameters

Power		
	Total thermal output	1980 MW_t
Core, reflector		
	Core radius	2.0 m
	Core height	2.0 m
	Reflector thickness (SS for Na coolant)	0.5 m
	Gas plenum length	1.0 m
Fuel cell		
	Fuel	Natural uranium metallic(75%TD)
	Cladding	ODS steel
	Coolant	Na or Pb-208
	Pin diameter	12.2 mm
	Cladding	0.5 mm
	Pin pitch	14.4 mm

Table 9-2 shows the optimum values of the adjusted parameters mentioned in Section 9.1 for both reactors. The sodium-cooled reactor requires less thorium addition than the lead-cooled reactor since it has a lower initial effective neutron multiplication factor (k_{eff}) (see Table 9-3). The sodium-cooled reactor is a much larger outer core thickness than the lead-cooled reactor since its neutron mean free path is larger.

Figure 9-2 shows the obtained power density distributions for both reactors; this figure also shows those for the original cores for comparison. The obtained effective neutron multiplication factors (k_{eff}) are shown in Table 9-3 before and after flattening for both reactors. Since the addition of thorium reduces the k_{eff} of the core, the value of k_{eff} before flattening is designed to be much higher than unity in both cases. Table 9-3 also shows the radial power peaking factor (the maximum to average ratios of the axially integrated power density). From these values, it is found that, for the given maximum axially integrated power density constraint, the total power can be increased by factors of 1.28 and 1.16 for the sodium-cooled and ^{238}Pb-cooled reactors, respectively. Although the improvement is larger for the sodium-cooled reactor, the ^{238}Pb-cooled reactor (whose radial peaking factor is almost unity) has a much flatter final power profile. This is attributed to the outer radial position of the boundary between the inner and outer cores, since the ^{238}Pb-cooled reactor has a smaller neutron mean free path.

Table 9-2. Outer Core Thickness and Thorium Addition Rate in Inner Core

	Na cooled	Pb-208 cooled
Th:U	22 : 78	37 : 63
Outer core thickness	120 cm	60 cm

Table 9-3. Obtained Effective Neutron Multiplication Factors and Radial Power Peaking Factors

	Na cooled		Pb-208 cooled	
	Original	Flattened	Original	Flattened
k_{eff}	1.015	1.000	1.043	1.002
Radial power peaking factor	1.815	1.416	1.231	1.063

The values obtained for the radial power peaking factor in Table 9-3 can be further improved by increasing the uranium density in the outer core gradually as a function of the radial position, such that the uranium density at the outer boundary is the maximum permissible value.

Figure 9-2 shows two-dimensional *r*–*z* power density distributions for both reactors before and after flattening. Power flattening straightens the axially dangling power density distribution. This effect is stronger for the ^{238}Pb-cooled reactor, and its core height can be shortened.

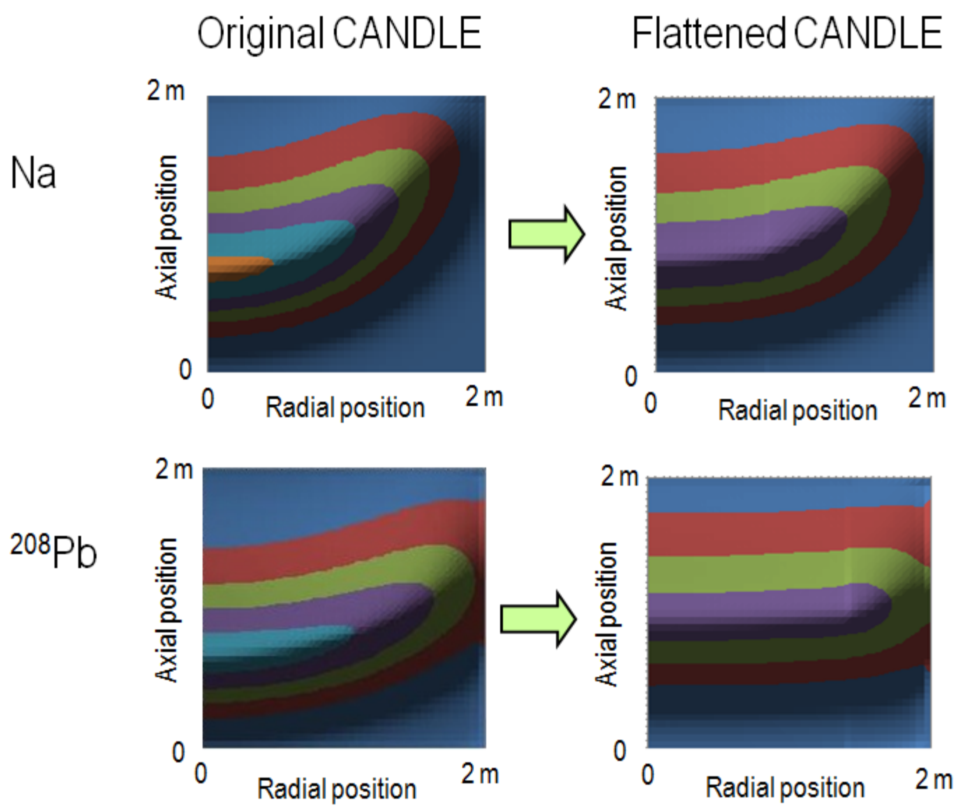


Figure 9-2. *r*–*z* power density distributions for two reactors before and after flattening.

The two-dimensional *r*–*z* power density distributions still exhibit distributions that curve in the axial direction in the outer core region. They can be straightened by employing the MOTTO cycle mentioned in Chapter 8.

10. History of Studies on CANDLE and Similar Burnups

This chapter presents a brief history of CANDLE burning and similar burning techniques. Section 10.1 gives a general history, while Section 10.2 gives a personal history of CANDLE study. The general history was written in 2008, at which time I was unaware of the new studies in TerraPower. Consequently, it does not include breed-and-burn reactors cited by TerraPower [Weaver et al., 2009], though one of the main topics in Section 10.2 is about TerraPower. However, I have included all papers that treat similar burning strategies that I found at that time.

The meeting between Bill Gates and Toshiba was reported on the front page of the Japanese newspaper *Nikkei* as the lead article on March 23, 2010. It resulted in a rush of media requests for interviews and articles on CANDLE reactors and travelling wave reactors. Among the requests, one was from the journal *Nuclear Viewpoints*. In response, I wrote a brief history of my research on CANDLE reactors commencing from my school days. Section 10.2 gives a translation of this article.

10.1. History of Studies on CANDLE and Similar Burnups up to 2008

The concept that the burning region moves with the progression of burnup might have been expected to appear relatively early in the history of nuclear reactor research, but it is difficult to find any mention of this concept in early papers. It is only recently that papers have been published on it. However, these papers have not been widely read and most of them were written independently of each other.

A preliminary study was performed in this field in Russia [Fomin et al., 2005]. The paper by Feoktistov in 1988 may be the first paper on this topic [Feoktistov, 1988]. He also published a paper [Feoktistov, 1989] in the next year. These papers were not distributed in the western world. I have not read these papers but expect that these studies may be based on an accelerator-driven system. The present book considers only stand-alone critical nuclear reactors.

Seifritz initially termed this burnup, solitary burnup wave, but he later used the term CANDLE. He performed many studies on this topic by introducing approximations to a one-dimensional model and treating it analytically [Seifritz, 1995, 1997, 1998, 2000, 2002a, 2002b, 2003, 2005, 2007, 2008]. Teller et al. performed a simulation of this kind of burnup in a thorium cylindrical core started by an external neutron source at the center [Teller et al., 1996; Hyde et al. 2008]. Van Dam named this burnup criticality wave and simulated it using a high-temperature gas-cooled reactor model [van Dam, 1998, 2003].

Sekimoto et al. formalized CANDLE burnup and developed a steady-state calculation code by employing the Galilean transformation [Sekimoto, Ryu, 2000a, Sekimoto et al. 2001a]. They investigated several fast reactor cores [Sekimoto, 2001, 2003, 2004, 2005a, b, 2006; Sekimoto, Nagata, 2008; Sekimoto, Tanaka, 2002a, b; Sekimoto, Yan, 2007; Sekimoto et al., 2001a, b; 2002a, b, 2003] and high-temperature gas-cooled reactor cores [Ohoka, Sekimoto, 2003, 2004a, b; Ohoka et al., 2004, 2005; Sekimoto, Ohoka 2003]. The spatial dependencies (equivalent to the burnup and temperature dependencies) of microscopic group constants were investigated and they demonstrated the good performance of this code [Sekimoto, 2004; Sekimoto, Udagawa, 2006]. Simulations were performed in the original time and space coordinates from the steady-state solution and from some initial cores constructed from readily obtainable materials [Ohoka, Sekimoto, 2004b; Sekimoto, Udagawa, 2005; Sekimoto, Miyashita, 2006; Sekimoto et al., 2003]

Recently, several scientists have become interested in this burning strategy and have published papers. Pilipenko et al. [Pilipenko et al., 2003] and Fomin et al. [Fomin et al., 2005, 2008] studied fast reactors. Chen and Maschek studied transverse buckling effects [Chen, Maschek, 2005]. Gaveau et al. studied an accelerator-driven system that employs a moving burning region [Gaveau et al., 2005, 2006].

Neutron-rich fast reactors can realize several innovative burnup strategies. Some reviews have been published, which include the CANDLE burnup strategy [Sekimoto et al., 2001b; Greenspan et al., 2003].

10.2. Personal History (translated from [Sekimoto, 2010b])

Upon opening my email client on the morning of March 23, 2010, I found a message from a friend who asked, “Have you seen the article in this morning’s *Nikkei* about Toshiba and Bill Gates? Isn't it about your reactor?" Our office secretary's first task that morning was to go out and get a copy of the morning edition. I thought I knew what had prompted the article. When a *Nikkei* reporter had visited my office a few days earlier regarding another matter, I had told him about an article on the Internet concerning a visit to China by Bill Gates. The Internet article mentioned near the end that he had also visited Toshiba. I assumed the reporter had then decided to visit Toshiba for some follow-up information, but when my secretary gave me the *Nikkei* morning edition, I found that the story was the lead article on the front page. Although it did not mention my name, the article immediately resulted in a rush of media requests for interviews and information, as it was obvious to many experts interested in innovative fast reactors that the one described in *Nikkei* was similar to the CANDLE reactor that is the focus of my research.

Among the requests, was one from the journal *Nuclear Viewpoints*. In previous media requests, I have often responded to questions about what the CANDLE reactor is, and I will not repeat that description here. My website also contains some technical information on the CANDLE reactor; I am planning to post an updated version of this on the CRINES center (where I serve as director) website <http://www.crines.titech.ac.jp> for readers interested in the technical and academic aspects of the reactor.

However, readers whose interest is more general than specialized invariably ask "When did you first conceive of the CANDLE concept?" and "Does this concept exist anywhere else?". I found that my answers were subjected to various forms of bias or distortion in many of the resulting articles, perhaps because it is relatively easy for others to expand on such questions. In this article, therefore, I describe the germination and growth of the CANDLE concept and the recent burgeoning interest in its background and potential.

1) Initial conception

The fact is I do not recall exactly when CANDLE burning concept first occurred to me. When I was studying fast reactors as an undergraduate student, I began wondering rather vaguely what would happen if a blanket fuel continued to burn, which involved questions about fuel burning methods. In relation to fast reactors, the first specialized book I read was “Fast Reactors” by Palmer and Platt; this also caused me to question and reflect on various types of fast reactors.

When that phase of my studies was almost complete, I attended the Summer Internship Program of the Japan Atomic Energy Research Institute (JAERI; now the Japan Atomic Energy Agency). There, I was asked to perform neutronics analyses for reactor cores. I was surprised that the computational code used in those analyses was apparently used in actual design work and that I could immediately perform the analyses myself. The diffusion equation was solved by Gaussian elimination. At university, I had already written a similar program for analyzing exponential experiments. Also, the ABBN set [Abagyan et al., 1964] was being used for group constants and I remembered that my thesis advisor had a book on ABBN on his office bookshelf. When I returned to the university campus, I told him about my thoughts, but he said there were more important things to do. For my Master’s thesis I went to the Kyoto University Research Reactor Institute, measured the energy spectra of neutrons leaking from a large iron assembly, and compared the obtained spectra with computational ones obtained using the neutron transport code that I had written.

My Master’s thesis project was a good experience but it had little to do with fast reactor design. I thought that when it was completed I might join JAERI. About that time, however, I was informed that the Power Reactor and Nuclear Fuel Development Corporation (PNC; now merged with the Japan Atomic Energy Agency) had just been formed to oversee the Japanese fast reactor program and that JAERI would no longer be working on fast reactors. When I inquired about opportunities at PNC, I learned that the type of nuclear reactor to be developed there had already been determined and that the envisioned program did not include research on the innovative nuclear reactors that I had in mind. Working in industry would apparently provide even less freedom. Ultimately, in accordance with my supervisor’s advice, I decided to pursue my studies in the US.

At the outset of my doctorate study in the US, my dissertation advisor, Prof. Pigford, and I talked about my crude idea of CANDLE burning when discussing which area of research to pursue. About 30 years later, when I met him again and talked about CANDLE burning, he spoke with fondness of the original discussion and I was surprised and quite moved to find that he had remembered it so clearly. He recalled our original discussion better than I did myself. Ultimately, I decided to do research on in-core fuel management of LWRs for my dissertation. At the time, interest in fast reactors was waning and there was the possibility that CANDLE burning might simply be impossible.

2) Subsequent twists and turns

After receiving my doctorate, I joined General Atomic Co. (GA), which was then deeply engaged in developing high-temperature gas reactor (HTGR) development. I felt that I would at last be able to focus on innovative reactors. I became engaged in work on spatial distribution stability of power density in large-scale reactors. GA generously granted me freedom in my research, but I felt that my proposal for innovative reactor research was a bit out of place. Before long, however, the HTGR project was drastically reduced. I began to feel a desire to pursue research on innovative reactors in a university setting and had the good fortune of being invited to work at Tokyo Institute of Technology (Tokyo Tech).

Initially, I was a research associate at Tokyo Tech and my research was focused on fusion neutronics in accordance with the needs of the laboratory. But when I was appointed associate professor, I soon decided to focus on innovative reactors. I thought that it might be difficult to join fast reactor communities as an experienced researcher if I immediately began to study such reactors, so I commenced researching HTGRs. My research experience at GA seemed to be highly regarded by Japanese HTGR communities. While I was at GA, I had proposed that an optimum power distribution could be obtained using fuel with just one kind of enrichment by causing the fuel to migrate in the axial direction. I discovered that this concept had already been developed in Germany as the pebble-bed reactor. Although my idea was not new, I realized that the computational method I had used in my doctoral dissertation could be effectively applied to the pebble-bed reactor. This work represented my first study in Japan directed toward researching innovative types of nuclear reactors.

About the time I was preparing to begin research on fast reactors, the Central Research Institute of Electric Power Industry (CRIEPI) launched with much fanfare its research program for the 4S reactor, a small fast reactor with metallic fuel. With my experience in research on the HTGR at GA, I initially felt it would be most difficult to achieve a practical small fast reactor, because of the obvious disadvantages of scale in the concept. But their explanation of the concept revealed that it actually did hold strong promise for ultimate practical realization, and I naturally became quite interested. By a fortunate turn of events, a student from Indonesia joined us at Tokyo Tech. This proved to be an excellent opportunity for researching small long-life reactors with the goal of constructing a reactor in Japan, transporting it to Indonesia where it would be operated, replacing it at the end of its life span with a new reactor built in Japan, and returning the used reactor to Japan.

The 4S Reactor was designed to utilize reflectors for burnup control, with the fuel burning region migrating from the bottom to the top of the reactor core. I felt the same thing should be possible without any burnup control methods such as the use of reflectors, if lead or lead-bismuth (lead bismuth eutectic; LBE) were used as the coolant rather than sodium for a metallic fuel fast reactor. With this in mind, we studied the question of whether CANDLE burning could be accomplished in an LBE-cooled small fast reactor. The study predicted complete failure. Everything would go wrong. I considered that it might work if the nuclear reactor was made larger, but in that case, the ultimate goal would be unachievable. However, I realized that if natural uranium was positioned in the central part of the core and burned from the circumference, the reactor could easily be operated continuously for more than 10 years. I therefore quickly gave up the original CANDLE burning configuration and decided to proceed with research on “out–in” burning in a small long-life fast reactor.

At this point, the development of the proposed LBE-cooled small long-life fast reactor took an unexpected twist. It was not long after the collapse of the Soviet Union, and I was invited to speak at a conference on small reactors being held near Moscow. When I presented the concept there, many in the audience had been involved in designing lead-bismuth-cooled beryllium-moderated small reactors for submarines and they became so interested by my concept that I was virtually mobbed by their questions and excitement. At that conference, there were no other reports on lead-bismuth small fast reactors and the concept apparently had not been proposed. But it was not long before they completed an excellent design, and through steady progress in research and development they are now about to commence construction.

3) Successful CANDLE burning computation

About the time when the design work at Tokyo Tech on the LBE-cooled metallic fuel small long-life fast reactor had been completed, a new student named Kouichi Ryu began studying there for his doctorate. As the timing seemed perfect, I decided to have him work on designing the CANDLE reactor using a LBE-cooled metallic fuel large fast reactor. However, at the same time, there were major doubts concerning the ultimate feasibility of the CANDLE reactor. For that reason, I first had the fast reactor designed as a pebble-bed reactor. The computational method had already been established with good results for the HTGR pebble-bed reactor, and I felt certain it would also be effective in this application. It did indeed give the expected results, and I had him complete a monograph on this research. When its acceptance for publication was assured, I next had him proceed to work on the CANDLE reactor. Only a slight portion of the computational code had to be modified for this purpose and several new innovations were necessary, but it resulted in the effective realization of CANDLE burning.

As Ryu had by then completed his dissertation and left the university, I compiled a paper on the validity of CANDLE burning with the help of Master students who had subsequently joined our laboratory. As I believed it would be epoch-making in its import, I submitted it to *Nuclear Science and Engineering* (NSE), the journal of the American Nuclear Society, which was at that time regarded as being the most authoritative journal by many of those engaged in nuclear energy research. The paper was submitted in 1999, but the referees apparently found the findings difficult to believe and requested various additional data, which were then incorporated so that the paper was not finally published until 2001 [Sekimoto et al., 2001a]. Following its submission, several applications were filed for presentations at international conferences and these presentations were actually given prior to the publication of the paper in NSE. When I spoke with an American friend concerning the content, he informed me of the publication of a similar concept by Edward Teller [Teller et al., 1996], which was ultimately added to the references in the NSE publication. The paper by Dr. Teller became the basis for the travelling wave reactor.

Sometime later, I was asked to give a presentation on the CANDLE reactor to a study group of senior members in the field of nuclear energy in Japan. Around that time, I had happened on the following interesting passage in a review column of *Nuclear News*: "The process demonstrated well the three stages of scientific discovery. In the first stage, people call your idea crazy and say you'd be a fool to propose it. In the second stage, with the evidence mounting, people say it just might be correct. By the time you have enough evidence to offer proof, you are at the third stage, when people say the idea is so obvious that of course it's true, any idiot could have seen that." At the group study meeting, I asked the listeners which stage they thought the CANDLE reactor was in. Everyone naturally replied that it was in the first stage. Sometime later, I happened to meet one of the participants and asked him again. Without hesitation, he replied that it was still in the first stage. Although this was not unexpected, I must admit that I found his response discouraging. Since that time, however, events have taken a new turn. Bill Gates has become part of the picture, and perhaps I should ask once again. The reply just might be that the CANDLE reactor is now in the second stage, although not on the strength of the technical findings but rather due to the financial prowess of Bill Gates. In any case, the third stage still appears a long way off.

In 2003, Tokyo Tech initiated its 21st Century COE program entitled "Innovative Nuclear Energy Systems for Sustainable Development of the World (COE-INES)" and selected the CANDLE reactor as one of its leading projects. The research will continue thereafter at the Center for Research into Innovative Nuclear Energy System (CRINES), as the COE successor.

4) The entry of Bill Gates

INES-2, the second COE-INES international symposium, was held in 2006. Edward Teller had passed away, but his co-researcher Lowell Wood was invited and we were honored by his acceptance and presentation [Hyde et al., 2008]. Though it had been some time since the publication of Dr. Teller's paper, Dr. Wood's presentation was essentially the same in content, which led me to wonder whether their work might have been a one-shot wonder. Sometime later, I received an email inviting me to visit Dr. Wood and his colleagues, not at Livermore as might be expected but rather at Seattle. Unfortunately, I was quite busy at the time and could not accept the invitation. In May 2009, however, I was introduced to John Gilleland, the CEO of TerraPower, at the ICAPP conference held in Tokyo that year, by my old friend Ning Li of Xiamen

University in China. I learned there that the nuclear reactor proposed by Dr. Teller and his colleagues had become the subject of research and development by Dr. Wood and others at TerraPower, as the TWR. I also learned that it was being sponsored by Bill Gates and that the development of a new reactor in China was envisioned. The organization as described to me seemed to have a deeply amateur quality, but I knew that any aspects in which it was lacking could be well remedied by the addition of outstanding personnel, and I felt that an organizational system of this nature might be needed to bring the CANDLE reactor to reality. I was invited to join as a consultant and was pleased to accept.

In the arrangements for the consultancy, the procedures at Tokyo Tech were quite lengthy and the formal signing was not completed until July of 2009. In September, I was able to schedule a visit to TerraPower in Bellevue, Washington. On my way there, I stopped in at my alma mater, the University of California, Berkeley, where I had been asked to speak about CANDLE. Although word had not yet reached Japan, the latest issue of *Nuclear News* contained a feature article on the TWR. Neither my name nor the name CANDLE appeared in the article, but people obviously knew that the CANDLE reactor and the TWR represented the same kind of nuclear reactor. The title of my speech at Berkeley was simply CANDLE, but the lecture hall was packed to overflowing.

The surroundings of the TerraPower research facility were luxurious, with towering trees and a deep green ambience. I was pleasantly surprised to find that not only Dr. Gilleland but also Dr. Wood and many researchers and graduates of MIT that I knew quite well were working at TerraPower. In the morning of the first day, I spoke to those who gathered there about the CANDLE reactor. Throughout the afternoon and the following day, they described to me the TerraPower organization and facilities, and the TWR.

In November, I received an urgent email message from my friend Ning Li asking me to come to China right away. He wrote that Bill Gates, Dr. Gilleland, and their associates had visited China in early November, and that he wanted me to visit the same institutions that they had. I hastened to adjust my schedule, and arrived in China at the end of November. I visited the China Institute of Atomic Energy (CIAE), the China National Nuclear Corporation (CNNC), the State Nuclear Power Technology

Corporation Ltd. (SNPTC), Tsinghua University, and the Shanghai Nuclear Engineering Research and Design Institute (SNERDI). At each location, Ning Li first spoke about the TWR and I then spoke about CANDLE. Many young people were in each audience. In the conference at CNNC in Beijing, I was surprised to meet one of my old students who was then working at a laboratory in Chengdu. He had simply been told to travel to Beijing for a seminar and had not even imagined that I would be speaking there.

It was at that time that I had received from Ning Li the Internet article about the visit of Bill Gates to China and also Toshiba.

5) Present status

Whenever I speak about CANDLE at international conferences, the lecture halls are filled to overflowing, whereas once the audiences were quite small or almost nonexistent. I think this may be partly due to the "Bill Gates effect". What I am doing now has changed very little from my previous work. It is true that many people have advised me to adopt a more attention-getting approach, but I think the present course is fine. I am most gladdened to see that innovative reactors are now becoming the focus of highly active research efforts. If the TWR someday proves successful, I will be delighted to know that many people are aware of its origins in the nuclear reactor named the CANDLE reactor that was first conceived and researched here in Japan.

Appendix

A1. Block-fuel HTGR

A1.1. Principle

The high-temperature gas-cooled reactor (HTGR) [Sekimoto et al., 2002] has attracted growing interest and various applications based on its use of high-temperature gas are envisioned. Lately, the high safety of the reactor has attracted attention and its excellent neutron economy has been recognized. As a result, construction of commercial reactors is planned. A further advantage of this reactor is that the integrity of coated fuel particles in the reactor can be maintained at a high burnup, and thus the reactor has attracted interest as a suitable reactor for eliminating plutonium and minor actinides. For details, see the explanation in Sekimoto et al., 2002.

HTGRs can be broadly classified into block-fuel reactors and pebble-bed reactors. Figure 2-6 shows schematic diagrams of these two types. Keep in mind, however, that the length ratios in these illustrations differ considerably from the actual length ratios; for example, the pebbles (fuel spheres) in the pebble-bed reactor are the size of tennis balls. The size of the pressure vessel in these nuclear reactors differs little from that of a large LWR. The driving mechanism for the control rods is illustrated only for the block-fuel reactor. For the pebble-bed reactor, only a control rod driving mechanism for start–stop control is necessary; control rods (shim rods) are not necessary for burnup control. Although the pebble-bed reactor has the advantage that refueling can be performed during operation, it has some technological complications.

For the application of CANDLE burning, the block-fuel HTGR is the most suitable nuclear reactor of the presently operated nuclear reactors since it does not require any drastic design changes [Ohoka, Sekimoto, 2004a]. Figure 2-7 shows the spatial distributions of the nuclide densities and neutron flux along the core axis.

In a thermal reactor, CANDLE burning is realized by adding burnable poison to the fuel. In Figure 2-7, gadolinium (Gd) is employed. When the microscopic absorption cross section of the burnable poison is sufficiently larger than that of the fissile material, the burnable poison will absorb neutrons leaking from the burning region to the fresh fuel region and will quickly disappear, as shown in Figure 2-7. Thus, fissile material remains in the fresh fuel region and the burning region can move into this region, realizing CANDLE burning. Burnable poisons are presently used in conventional nuclear reactors for suppressing excess reactivity during burning. Thus, the self-shielding effect, which alters the neutron absorption rate, is conveniently utilized. However, in CANDLE burning, the burnable poison ideally disappears as soon as possible. Thus, it is thinly mixed into a graphite matrix to reduce self-shielding.

As is clear from Figures 2-4 and 2-6, CANDLE-type refueling is possible for block fuel without drastic design changes, unlike the pin-type fuel in LWRs. Note that in Figures 2-4, to emphasize the characteristics of CANDLE, the moving distance of the burning region is shown to be long. Hence, the figure is quite different from the actual design as it shows the exchange section as being large. In reality, one block of spent fuel is removed and one block of fresh fuel loaded. Even in this case, the lifetime of an operation cycle is usually a few years.

A1.2. Advantages

Applying CANDLE burning to a block-fuel HTGR has the following advantages.

1) It shares the major advantages of the pebble-bed reactor.
   - It does not require control rods for burning control.
     This is very important from a safety viewpoint, so I will explain it further. In a HTGR, the coolant helium pressure is high, around 70 atmospheres. Therefore, the driving mechanism of the control rods, which runs through the pressure vessel, may jump out. If this happens, the reactivity of the nuclear reactor will suddenly increase greatly and the power may run out of control. In a CANDLE reactor, there are no control rods for burning control.
   - The characteristics of the nuclear reactor do not change with time.
   - Operation is simple and highly reliable.
2) It has additional advantages to those of the pebble-bed reactor.

- Complicated equipment used for on-power refueling is not necessary.
- In the pebble-bed reactor, the burning history of each pebble varies randomly; thus, it is uncontrollable and unpredictable. In a CANDLE core, the burning of each element of fuel is controllable and predictable.
- In the pebble-bed reactor, fuel pebbles pile up and move. Therefore, they may be damaged. This does not occur in a block fuel reactor.

3) The maximum fuel temperature can be lowered by channeling the coolant in the opposite direction of the movement of the burning region.

The power distribution shifts in the direction of the movement of the burning region and exponentially decreases in the opposite direction. For such a power distribution, the maximum fuel temperature can be reduced by channeling the coolant in the opposite direction to the movement of the burning region.

The other advantages described in Section 2.2 can also be achieved.

A1.3. Analysis Results

Table A1-1 shows the design parameters for an example block-fuel HTGR. Natural gadolinium is used as burnable poison. For the thermal output and core shape, the values for the High-Temperature Engineering Test Reactor (HTTR), operated at JAERI, were mostly adopted. HTTR is an experimental reactor and its thermal output is extremely low. Thus, this design is not suitable for a commercial reactor. However, the values for this reactor were adopted since the design data are readily available.

For the calculation, a four-group diffusion equation, which is often used for the analysis of HTGR, was used. The group constants were obtained using the SRAC code system [Okumura, et al., 1996] with the JENDL-3.2 library [Shibata, et al., 2002]. Because of the restriction of the code, there was no option other than to mix the burnable poison with the fuel kernel. This causes the microscopic cross section of the burnable poison to become small because of the neutron shielding effect, and the CANDLE characteristics deteriorate.

Table A1-1. Design parameters of a block-fuel HTGR.

Reactor	Thermal output	30 MWt
	Core radius	115 cm
	Radial reflector thickness	100 cm
Fuel	^{235}U enrichment	15%
	Fuel kernel	UO_2
	Burnable poison	Natural Gd (3.0%)
	Cladding	TRISO
	Kernel diameter	0.608 mm
	Coated fuel particle diameter	0.940 mm
	Particle packing factor	30%

Table A2-2. Calculation results for a block-fuel HTGR.

Effective neutron multiplication factor		1.008
Moving speed of burning region		29.2 cm/y
Axial half width of power density		154 cm
Burnup	Maximum	12.3%(115.2 GWd/t)
	Average	10.7%(100.3 GWd/t)

These calculations confirm that CANDLE burnup is realized for this design. The results are shown in Table A2-2. The burnup was small, although it was much larger than the HTTR value, and it cannot be claimed that the results were good. However, this is due to the fact that the burnable poison had to be put into the fuel kernel. In the future, the burnable poison will be mixed with graphite, which will drastically improve the results. Thus, there is no technological problem to solve.

We have investigated using this method to eliminate surplus [Ohoka, Sekimoto, 2003b]. The higher the burnup is, the better CANDLE burnup is achieved. It was shown that about 90% of ^{239}Pu can be eliminated. If the burnup is increased in a conventional reactor, the power distribution will be greatly distorted. In addition, characteristics such as the reactivity coefficient change drastically with the burnup. On the other hand, a CANDLE reactor shows an unchanged smooth distribution and unchanged reactivity coefficients even at very high burnup.

A2. Technical Terms

These technical terms have been compiled only to assist in reading this book. Hence, the explanations of the terms may not necessarily be entirely precise.

Note on expression of atomic nucleus: To fully express an atomic nucleus, the atomic symbol is written with the atomic number (number of protons) written as subscript on the left and the mass number (total number of protons and neutrons) written as superscript also on the left side. Once the atomic symbol is determined, the atomic number can be uniquely determined, and thus the atomic number is often omitted. For example, ^{235}U is written for uranium-235.

Burnable poison: Neutron absorber that is inserted into the core. The neutron absorber with large microscopic absorption cross section is converted, with the progress of burnup, into a material with a small neutron absorption cross section. It is used to lessen the reduction of the effective neutron multiplication factor in the early stage of burnup.

Burnup: It has two meaning in this booklet. One is the change of fissile material into fission products through nuclear fission in a reactor core. The other is the generated energy per unit of spent fuel. The unit GWd/t is usually used. This expresses generated energy in GWd (giga-Watt days), per weight of uranium and plutonium in t (tons) contained in fresh fuel. Sometimes the expression is given in %.

Burnup reactivity: The same as excess reactivity. (See excess reactivity.)

Cladding tube: A tube that covers fuel pellets to prevent a leak of radioactive material from the fuel into the coolant and other elements of the reactor.

Coated fuel particles: Fuel particles of about 1 mm diameter used in a HTGR. Fuel kernels are coated with graphite and silicon carbide.

Control rods: The criticality of a nuclear reactor is adjusted with these rods, which are made of neutron absorber. The power level and shape can also be adjusted. They are also effective in stopping the operation of a nuclear reactor.

Core: Region where fuel is located in a nuclear reactor.

Criticality: A state in which neutrons stays under the balance of generated neutrons and consumed (absorbed or leaked) neutrons. If the number of generated neutrons is larger than the number of consumed neutrons, the state is called supercritical. If the number of consumed neutrons is larger, it is called subcritical.

Criticality experiment: An experiment to verify the precision of calculations by assembling fuel, achieving criticality, and comparing the critical amount of fuel and other measurements with the calculated values.

Cross section: The probability that a nuclear reaction takes place. The larger the cross section, the more likely that a nuclear reaction will take place. (See microscopic absorption cross section.)

Decay: See radioactive decay.

Depleted uranium: When natural uranium is enriched to obtain enriched uranium, a large amount of uranium containing less ^{235}U than natural uranium is generated. This is called depleted uranium.

Effective neutron multiplication factor: Neutron multiplication factor for an actual core under consideration. If the core in consideration is critical, the factor is exactly unity. If it is subcritical, the factor is less than unity, and if supercritical, it is more than unity. (See neutron multiplication factor.)

Excess reactivity: In a normal nuclear reactor, the reactivity at the start of burnup is positive. However, the reactivity becomes smaller with the progress of burnup. When the reactivity becomes zero, the operation is stopped and refueling is required to continue the operation. The reactivity is suppressed with control rods to attain criticality. The reactivity described above is called excess reactivity.

Fast reactor: A nuclear reactor in which neutrons are not moderated and the nuclear fissions are caused by fast neutrons. Water, which moderates neutrons, cannot be used as a coolant. Thus, sodium, lead (or lead-bismuth eutectic), or gas is used as a coolant.

Fertile material: Material that does not undergo nuclear fission when a thermal neutron is absorbed, but instead becomes fissile material.

Fissile material: Material that fissions by the absorption of a thermal neutron. Fissile material does not necessarily fission after absorbing a neutron and to distinguish the absorption of a neutron without nuclear fission, it is called capture.

Fission products: When fissile material undergoes nuclear fission, two fission products are generated in most cases. Nuclear fission does not take place when a neutron is absorbed by a fission product.

Fuel cycle: Fuel cycle is generally a stream of fuel in a nuclear energy utilization system with nuclear reactors, but in this booklet it means the following specific fuel cycle. Fuel from a nuclear reactor is reprocessed, fissile material is separated and processed into fuel, and is then returned to the nuclear reactor. This is the cycle of fuel. Nowadays, however, the fuel cycle includes the mining of uranium to the final disposal of waste.

Fuel kernel: Fuel sphere located at the center of coated fuel particle. (See coated fuel particles.)

Galilean transformation: Transformation from one coordinate system to another coordinate system that is moving at a different speed. In the case considered in this book, the two coordinate systems are one at rest and one traveling at a speed V.

Half life: Time necessary for a radioactive material to decay to half of its original amount.

HTGR: A reactor in which graphite is used for to moderate neutrons and high temperature helium is used for cooling. Fuel is prepared by mixing coated fuel particles into graphite.

Infinite medium neutron multiplication factor: The neutron multiplication factor where the size of the core is assumed to be infinite. This is expressed in k_∞. (See neutron multiplication factor.)

Light-water reactor: A reactor in which light water (normal water, as distinguished from heavy water) is used to moderate neutrons and to cool the core. Presently, most extensively operated reactors are light-water reactors. In a boiling-water reactor (BWR), water boils in the core, and in a pressurized-water reactor (PWR), water does not boil in the core.

Microscopic absorption cross section: Neutron absorption cross section per nucleus. (See cross section.)

Neutron fluence: Time-integrated neutron flux of particles per unit area. (See neutron flux.)

Neutron flux: A quantity obtained by multiplying the neutron density and the neutron speed. The reaction rate is obtained by multiplying the neutron flux and the cross section.

Neutron spectrum: Energy distribution of neutrons.

Neutron multiplication factor: The rate of change of the average number of neutrons during one cycle. Here, one cycle is from one nuclear fission to the succeeding nuclear fission. In a critical state it is exactly unity, in a subcritical state it is less than unity, and in a supercritical state it is more than unity. (See effective neutron multiplication factor and infinite medium neutron multiplication factor.)

Nuclear proliferation: The spread of nuclear weapons to countries or organizations whose possession of the weapons is not approved.

Nuclear proliferation resistance: Deterrence of nuclear proliferation.

Nucleons: Particles that constitute a nucleus, namely, protons and neutrons.

Nuclide: Species of atomic nuclei. A nuclide can be uniquely determined once the number of protons and the number of neutrons in the nucleus are determined.

Once-through: Spent fuel is permanently disposed of as is.

Peaking factor: Ratio between the maximum value and the average value of power density.

Power/thermal power: Power of a nuclear reactor. The unit used is watt. Since the values are big, MW (mega-watt; mega means 10^6) is used. When burnup is expressed, GW (giga-watt; giga means 10^9) is often used. Power may be thermal or electric. If the efficiency of power generation is known, the electric power can be calculated from the thermal power.

Power coefficient of reactivity: The change in reactivity due to a change in power. In a normal nuclear reactor, the value should be negative so that the nuclear reactor can be stably controlled. If the value is positive, there is a possibility that the power will go out of control because of control instability. (See reactivity.)

Radioactive decay: A change into another nuclide through radiation. Typical decays are α-decay, which releases a nucleus of helium (α-ray), β-decay, which releases an electron (β-ray), and γ-decay, which releases high energy electromagnetic waves (γ -ray).

Reactivity: A value that indicates how far away the effective neutron multiplication factor is from criticality. If the effective neutron multiplication factor is expressed by k, the reactivity is defined as $(1-k)/k$. (See effective neutron multiplication factor.)

Reactor physics: The study that deals with neutron behavior in a nuclear reactor, where the criticality characteristics, power distribution, power coefficient of reactivity, etc. are analyzed.

Reflector: A component that returns leaking neutrons from the core back to the core. (See core.)

Reprocessing: Extraction of fissile materials, especially plutonium, from spent fuel, and associating processes.

Thermal neutron: Neutrons generated by nuclear fission have high energy. Thermal neutrons are obtained by moderating these neutrons and decreasing their energy to the same level as the temperature of the medium. Generally the probability of nuclear reaction with thermal neutrons is much higher than with fast neutrons.

References

Abagyan, et al., 1964. Group Constants for Nuclear Reactor Calculations, Consultants Bureau, New York.

Buongiorno, J., Todreas, N., Kazimi, M., 1999. Thermal Design of Lead-Bismuth Cooled Reactors for Actinide Burning and Power Production, MIT-ANP-TR-066, Massachusetts Institute of Technology, Cambridge Massachusetts.

Chen, X., Maschek, W., 2005. Transverse buckling effects on solitary burn-up wave, Ann. Nucl. Energy, 32, p. 1377-1390.

Chen, X., Kiefhaber, E., Maschek, W., 2008. Fundamental Burn-up Solution for Pebble-Bed Type Reactor, Progress Nucl. Energy, 50, pp. 219-224.

Choi, H., et al., 2001. Economic Analysis on Direct Use of Spent Pressurized Water Reactor Fuel in CANDU Reactors - II: DUPIC Fuel-Handling Cost, Nucl. Technol., 134, pp. 130-148.

Clark, L., Hansen, K., 1964. Numerical Methods of Reactor Analysis, Academic Press, New York.

van Dam, H., 1998. Burnup waves, Ann. Nucl. Energy, 25, p. 1409-1417.

van Dam, H., 2000. Self-stabilizing criticality waves, Ann. Nucl. Energy, 27, p. 1505-1521.

van Dam, H., 2003. Flux distributions in stable criticality waves, Ann. Nucl. Energy, 30, p. 1495-1504.

Duderstadt, J., Hamilton, L., 1976. Nuclear Reactor Analysis, John Wiley & Sons, Inc., New York.

Dupree, S., F. Stanley, 2002. A Monte Carlo Primer, Kluwer Academic/Plenum Publishers, New York.

Feoktistov, L. P., 1988. An Analysis of a Concept of a Physically Safe Reactor, Preprint IAE-4605/4, 1988. (in Russian).

Feoktistov, L.P., 1989. Neutron-fissioning wave, Dokl. Akad. Nauk SSSR, 1989, Vol. 309, pp 864–867. (in Russian).

Fomin, S., Mel'nik, Y., Pilipenko, V., Shul'ga, N., 2005. Investigation of self-organization of the non-linear nuclear burning regime in fast neutron reactors, Ann. Nucl. Energy, 32, pp. 1435-1456.

Fomin, S., Mel'nik, Y., Pilipenko, V., Shul'ga, N., 2008. Initiation and Propagation of Nuclear Burning Wave in Fast Reactor, Progress Nucl. Energy, 50, pp. 163-169.

Gaveau, B., Maillard, J., Maurel, G. Silvia, J., B., 2005. Hybrid soliton nuclear reactors: A model and simulation, Nucl. Engin. Design. 235, pp. 1665-1674.

Gaveau, B., Maillard, J., Maurel, G. Silvia, J., 2006. Equations of evolution of a hybrid soliton nuclear reactor, J. Math. Pure Appl., 85, pp 633-670.

Gilleland, J., Ahlfeld, C., Dadiomov, D., Hyde, R., Ishikawa, Y., McAlees, D., McWhirter, J., Myhrvold, N., Nuckolls, J., Odedra, A., Weaver, K., Whitmer, C., Wood, L., Zimmerman, G., 2008. Novel Reactor Designs to Burn Non-Fissile Fuels, Proc. of International Congress on Advances in Nuclear Power Plants (ICAPP '08), Anaheim, California.

Goldstein, H., 1950. Classical Mechanics, Addison-Wesley Publishing Co., Reading, Massachusetts.

Greenspan, E., Hejzlar, P., Sekimoto, H., Toshinsky, G., Wade, D., 2003. New Fuel Cycle and Fuel Management Options in Heavy Liquid Metal-Cooled Reactors, Nucl. Technol., 151[2], pp. 177-191.

Gromov, B., et al., 1997. Use of lead-bismuth coolant in nuclear reactors and accelerator-driven systems, Nucl. Eng. Design, 173, pp. 207-217.

Hiraoka, T., et al., 1991. A High-Breeding Fast Reactor with Fission Product Gas Purge/Tube-in-Shell Metallic Fuel Assemblies, Nucl. Technol., 93, pp. 305-329.

Hyde, R., Ishikawa, M., Myhrvold, N., Nuckolls, J., Wood, L., 2008. Nuclear Fission Power for 21st Century Needs: Enabling Technologies For Large-Scale, Low-Risk, Affordable Nuclear Electricity, Progress Nucl. Energy, 50, pp. 82-91.

Ismail, Liem, P. Takaki, N. Sekimoto, H., 2006. Systems of Symbiotic Large FBRs and Small CANDLE-Thorium-HTGRs, PHYSOR-2006, Vancouver, Canada.

Ismail, Ohoka, Y., Liem, P., Sekimoto, H., 2007. Long Life Small CANDLE-HTGRs with Thorium, Ann. Nucl. Energy, 34[1-2], pp. 120-129.

Khorasanov, G., et al., 2009. Minimization of initial fast reactor uranium–plutonium load by using enriched lead-208 as a coolant, Nucl. Eng. Des., 239, p. 1703.

Nagata, A., Sekimoto, H., 2007a. Analysis of Recladding in CANDLE Reactor, 15th International Conference on Nuclear Engineering (ICONE-15), Nagoya, Japan.

Nagata, A., Sekimoto, H., 2007b. Comparison of Removed Fuel Compositions of CANDLE, PWR and FBR, GLOBAL 2007 Advanced Nuclear Fuel Cycles and Systems, Idaho.

Nagata, A., Takaki, N., Sekimoto, H., 2009. A feasible core design of lead bismuth eutectic cooled CANDLE fast reactor, Ann. Nucl. Energy, 36[5], pp. 562-566.

Nakagawa, T., et al., 1995. Japanese Evaluated Nuclear Data Library Version 3 Revision-2: JENDLE-3.2, J. Nucl. Sci. Technol., 32, p. 1259.

Obara, T., Sekimoto, H., 1991. New Numerical Method for Equilibrium Cycle of High Conversion Pebble Bed Reactors, J. Nucl. Sci. Technol., 28[10], pp. 947-957.

Ohoka, Y., Sekimoto, H., 2003a. Application of CANDLE Burnup to Block-Type High Temperature Gas Cooled Reactor, ICONE11, Tokyo, Japan.

Ohoka, Y., Sekimoto, H., 2003b. Application of CANDLE Burnup to Block-Type High Temperature Gas Cooled Reactor for Incinerating Weapon Grade Plutonium, GENES4/ANP2003, Kyoto, JAPAN.

Ohoka, Y., Sekimoto, H., 2004a. Application of CANDLE Burnup to Block-type High Temperature Gas Cooled Reactor; Nucl. Engin. Design, 229[1], pp. 15-23.

Ohoka, Y., Sekimoto, H., 2004b. Simulation Study on CANDLE Burnup of High Temperature Gas Reactor, Trans. Ameri. Nucl. Society, 92.

Ohoka, Y., Ismail, Sekimoto, H., 2004. Effects of Burnup and Temperature Distributions to CANDLE Burnup of Block-Type High Temperature Gas Cooled Reactor; ICAPP '04, Pittsburgh, PA.

Ohoka, Y., Sekimoto, H., Watanabe, T. Liem, P.H., Wakana, S., Ismail, 2005a. Neutronic Characteristics of CANDLE Burnup Applied to Block-Type High Temperature Gas Cooled Reactor, ICAPP'05, Seoul, Korea.

Okawa, T., Nakayama, S., Sekimoto, H., 2010. Design Study on Power Flattening to Sodium Cooled Large-Scale Candle Burning Reactor with Using Thorium Fuel, Proc. of the 18th International Conference on Nuclear Engineering (ICONE18), Xi'an, China.

Okumura, K., et al., 1996. SRAC95; General Purpose Neutronics Code System, JAERI-Data/Code 96-015, Japan Atomic Energy Research Institute.

Pilipenko, V., Belozorov, D., Davydov, L. & N. Shul'ga, 2003. Some Aspects of Slow Nuclear Burning, ICAPP'03, Cordoba, Spain.

Ronen, Y., 1986. Handbook of Nuclear Reactors Calculations, vol. 1, CRC Press, Inc., Boca Raton.

Seifritz, W., 1995. Non-Linear Burn-up Waves in Opaque Neutron Absorbers, Kerntechnik 60, p. 185.

Seifritz, W., 1997. The nuclear soliton reactor, Kerntechnik 60, pp. 185-188.

Seifritz, W., 1998. On the burn-up theory of fast soliton reactors, Int. J. Hydrogen Energy, 23, p. 77.

Seifritz, W., 2000. The solitary burn-up waves in a multiplying medium, Kerntechnik 65, pp. 261-264.

Seifritz, W., 2002a. Approximation of solitary burn-up waves by generalized Gomperts functions, Kerntechnik 67, p. 130.

Seifritz, W., 2002b. Completer integration of the non-linear neutron flux wave equation in a purely absorbing medium, Kerntechnik 67, p. 130.

Seifritz, W., 2003. Format's optics in nuclear materials, Kerntechnik 68, p. 262.

Seifritz, W., 2005. On the plane-wave theory in fertile uranium, Kerntechnik 70, p. 309.

Seifritz, W., 2007. Observation on the CANDLE Burn-up in Various Geometries, 13th International Conference on Emerging Nuclear Energy Systems (ICENES'2007), Istanbul, Turkey..

Seifritz, W., 2008. The analytical representation of the fundamental mode in 1-D-geometry for the CANDLE burn-up phenomenon, Kerntechnik 73, pp. 1-2.

Sekimoto, H., Pigford, T. H., 1974. A New Method for Calculating Space-Dependent Burnup, Trans. Amer. Nucl. Society, 18, p. 137.

Sekimoto, H., Obara, T., Yukinori, S., Suetomi, E., 1987. New Method to Analyze Equilibrium Cycle of Pebble-Bed Reactors, J. Nucl. Sci. Technol., 24, pp. 765-772.

Sekimoto, H., 1992. Several Features and Applications of Small Reactors, Potential of Small Nuclear Reactors for Future Clean and Safe Energy Sources, Elsevier, Amsterdam, Netherlands, pp. 23-32.

Sekimoto, H., Zaki S., 1995. Design Study of Lead- and Lead-Bismuth-Cooled Small Long-Life Nuclear Power Reactors Using Metallic and Nitride Fuel, Nucl. Technol., 109, p. 307.

Sekimoto, H., Ryu, K., 2000a. A New Reactor Burnup Concept “CANDLE”, PHYSOR 2000, Pittsburgh.

Sekimoto, H., Ryu, K., 2000b. Feasibility Study on the CANDLE New Burnup Strategy, Trans. American Nuclear Society, 82, pp. 207-208.

Sekimoto, H., Ryu, K., 2000c. A Long Life Lead-Bismuth Cooled Reactor with CANDLE Burnup, ICENES 2000, Petten, Netherlands, pp. 198-206.

Sekimoto, H., Ryu, K., 2000d. Demonstrating the Feasibility of the CANDLE Burnup Scheme for Fast Reactors, Trans. Amer. Nucl. Society, 83, p. 45.

Sekimoto, H., 2001. Applications of “CANDLE” Burnup Strategy to Several Reactors, ARWIF-2001, Chester, UK.

Sekimoto, H., Ryu, K., Yoshimura, Y., 2001a. CANDLE: The New Burnup Strategy, Nucl. Sci. Engin., 139[3], pp. 306-317.

Sekimoto, H., Toshinsky, V., Ryu, K., 2001b. Natural Uranium Utilization without Enrichment and Reprocessing, GLOBAL 2001, Paris, France.

Sekimoto, H., Tanaka, K., 2002a. CANDLE Burnup for Different Core Designs, PHYSOR2002, Seoul, Korea.

Sekimoto, H., Tanaka, K., 2002b. Application of CANDLE Burnup Strategy to Small Reactors, Trans. Amer. Nucl. Society, 87.

Sekimoto, H., et al., 2002. New Deployment of High Temperature Gas-Cooled Reactor, J. Atm. Energy Soc. of Japan, 44, pp.839-866 (written in Japanese)

Sekimoto, H., 2003. Contribution of CANDLE Burnup to Future Equilibrium Nuclear Energy Utilization, GLOBAL 2003, New Orleans, Louisiana.

Sekimoto, H., Y. Ohoka, 2003. Application of CANDLE Burnup to Block-Fuel High Temperature Gas Reactor, ICAPP’03, Cordoba, Spain.

Sekimoto, H., Takada, T., Udagawa, Y., 2003. Startup of CANDLE Burnup in an LBE-Cooled Metallic Fuel Fast Reactor, GLOBAL 2003, New Orleans, Louisiana.

Sekimoto, H., 2004. Effect of Neutron Spectra and Fuel Burnup on CANDLE Calculation, Trans. Ameri. Nucl. Society, 92.

Sekimoto, H., Ohoka, Y., 2004. Burnup and Temperature Effects on CANDLE Burnup of Block-Type High Temperature Gas Cooled Reactor, ICONE12, Arlington, Virginia.

Sekimoto, H., 2005a. Application of “CANDLE” Burnup to LBE Cooled Fast Reactor, IAEA-TECDOC-1451, pp. 203-212.

Sekimoto, H., 2005b. Application of CANDLE Burnup Strategy for Future Nuclear Energy Utilization, Progress Nucl. Energy, 47[1-4], pp. 91-98.

Sekimoto, H., Udagawa, Y., 2005. Shut-down and Restart Simulation of CANDLE Fast Reactors, Trans. Amer. Nucl. Society, 93.

Sekimoto, H., Miyashita, S., 2005. Startup of “Candle” Burnup in Fast Reactor from Enriched Uranium Core, ICENES’2005, Brussels, Belgium.

Sekimoto, H., Udagawa, Y., Ohoka, Y., 2005. Application of “CANDLE” Burnup to Fast and Thermal Reactors, ICAPP ’05, Seoul, Korea.

Sekimoto, H., 2006a. Fuel-cycle of CANDLE Burnup with Depleted Uranium, ICAPP’06, Reno, Nevada.

Sekimoto, H., 2006b. Candle Burnup in a Fast Reactor Core and Relating Nolinear Prblems, 2nd International Conf. on Quantum Electrodynamics and Statistical Physics (QEDSP2006), Kharkov Ukraine.

Sekimoto, H., Udagawa, Y., 2006. Effects of Fuel and Coolant Temperatures and Neutron Fluence on CANDLE Burnup Calculation, J. of Nucl. Sci. Technol., 43[2], pp. 189-197.

Sekimoto, H., Miyashita, S., 2006. Startup of “Candle” Burnup in Fast Reactor from Enriched Uranium Core, Energy Conv. Manag., 47[17], pp. 2772-2780.

Sekimoto, H., Yan, M., 2007. Design Study on Small Candle Reactor, 13th International Conference on Emerging Nuclear Energy Systems (ICENES’2007), Istanbul, Turkey.

Sekimoto, H., 2008a. Basic Theory of CANDLE Burn-up, Proc. of International Conference on the Physics of Reactors “Nuclear Power: A Sustainable Resource”, (PHYSOR08), Interlaken, Switzerland.

Sekimoto, H., 2008b. Practical Design of CANDLE Reactor, INSAC-2008: Nuclear Technology- Role of Industry, Mumbai, India.

Sekimoto, H. Nagata, A., 2008a. Fuel Cycle for “CANDLE” Reactors, Proc. of Workshop on Advanced Reactors With Innovative Fuels ARWIF-2008, Tsuruga/Fukui, Japan.

Sekimoto, H., Nagata, A., 2008b. “CANDLE” burnup regime after LWR regime, Progress Nucl. Energy, 50, pp. 109-113.

Sekimoto, H., Yan, M., 2008a. A Design and Safety Features of Small CANDLE Fast Reactor, Proc. of International Congress on Advances in Nuclear Power Plants (ICAPP ’08), Anaheim, California.

Sekimoto, H., Yan, M., 2008b. Design study on small CANDLE reactor, Energy Conversion and Management, 49[7], pp. 1868-1872.

Sekimoto, H., 2009a. Five Requirements for Nuclear Energy and CANDLE Reactors, Materials for Heavy Liquid Metal Coolant and Related Technologies (HeLiMeRT 2009), Jeju, Korea.

Sekimoto, H., 2009b. CANDLE Reactor: An option for simple, safe, high nuclear proliferation resistant, small waste and efficient fuel use reactor, Nuclear Power and Energy Security (ed. S. Apikyan and D. J. Diamond), pp. 197- 204, Springer.

Sekimoto, H., Nagata, A., 2009a. Core Height Shortening of CANDLE Reactor by Employing MOTTO Cycle, Trans. American Nuclear Society, 101 (CD).

Sekimoto, H., Nagata, A., 2009b. Optimization of Power Profile of CANDLE Reactor by Using MOTTO Cycle, 14th International Conference on Emerging Nuclear Energy Systems (ICENES2009), Ericeira, Portugal.

Sekimoto, H., 2010a. Six Requirements for Nuclear Energy System and Candle Reactor, European Nuclear Conference (ENC 2010), Barcelona, Spain.

Sekimoto, H., 2010b. The rise of interest in the CANDLE reactor, Nuclear Viewpoints, vol. 56[7], pp.8-11.

Sekimoto, H., Nagata, A., 2010. Introduction of MOTTO Cycle to CANDLE Fast Reactor, PHYSOR 2010 - Advances in Reactor to Power the Nuclear Renaissance, Pittsburgh, Pennsylvania.

Sekimoto, H., Nakayama, S., Taguchi, H., Ohkawa, T., 2010a. Power Flattening for Sodium Cooled Metallic Fuel CANDLE Reactor by Adding Thorium in Inner Core, PHYSOR 2010 - Advances in Reactor to Power the Nuclear Renaissance, Pittsburgh, Pennsylvania.

Sekimoto, H., Nakayama, S., Taguchi, H., Ohkawa, T., 2010b. Power Flattening for CANDLE Fast Reactor by Adding Thorium in Inner Core, Proc. of ICAPP’10, San Diego.

Shibata, K, et al., 2002. Japanese Evaluated Nuclear Data Library Version 3 Revision-3: JENDL-3.3, J. Nucl. Sci. Technol. 39, p. 1125.

Tachihara, H., Sekimoto, H., 1999. Exact Error Estimation for Solutions of Nuclide Chain Equations, J. Nucl. Sci. Technol., 36, pp. 1176-1185.

Takaki, N., Sekimoto, H., 2008. Potential of CANDLE reactor on sustainable development and strengthened proliferation resistance, Progress Nucl. Energy, 50, pp. 114-118.

Teller, E., Ishikawa, M., Wood, L., Hyde, R., Nuckolls, J., 1996. Completely Automated Nuclear Reactors for Long-Term Operation II, ICENES'96, Obninsk, Russa.

Weaver, K., Ahlfeld, C., Gilleland, J., Whitmer, C., Zimmerman, G., 2009. Extending the Nuclear Fuel Cycle with Traveling-Wave Reactors, GLOBAL 2009, Paris, France.

Yan, M. Sekimoto, H., 2007a. Small Long Life CANDLE Fast Reactor Research (Part II: Accident Analysis), PHYTRA1, Marrakech, Morocco.

Yan, M., Sekimoto, H., 2007b. Small Long Life CANDLE Fast Reactor Research, 15th International Conference on Nuclear Engineering (ICONE-15), Nagoya, Japan.

Yan, M., Sekimoto, H., 2008a. Design research of small long life CANDLE fast reactor, Ann. Nucl. Energy, 35[1], pp. 18–36.

Yan, M., Sekimoto, H., 2008b. Study on small long-life LBE cooled fast reactor with CANDLE burn-up – Part I: Steady state research, Progress Nucl. Energy, 50, pp. 286-289.

Yan, M., Sekimoto, H., 2008c. Safety analysis of small long life CANDLE fast reactor, Ann. Nucl. Energy, 35[5], pp. 813–8285.

Zaki S., Sekimoto, H., 1992. A Concept of Long-Life Small Safe Reactor, Potential of Small Nuclear Reactors for Future Clean and Safe Energy Sources, Elsevier, Amsterdam, the Netherlands, pp. 225-234.