APPLICATION OF INFORMATION THEORY TO REACTOR PHYSICS

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SCOPE OF THE TALK

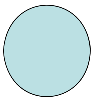
- **QUALITATIVE INTRODUCTION TO INFORMATION THEORY.**
- **CONSTRUCTION OF COVARIANCE MATRIX.**
- **REDUCTION OF SYSTEMATIC UNCERTAINTY.**
- **APPLICATION TO REACTOR PHYSICS.**

INFORMATION AGE

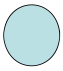
- **ABUNDANT INFORMATION.**
- **INTERNET.**
- **INFORMATION AND UNCERTAINTY.**
- **UNCERTAINTY =**
- **[1 ／ INFORMATION]**

COMMUNICATION THEORY.

- **INPUT SIGNAL**
- **OUTPUT SIGNAL.**
- **FOR ZERO NOISE.**



- **FOR NON ZERO NOISE**



COMMUNICATION THEORY

- **OUT SIGNAL = [IN SIGNAL] ／ [NOISE]**
- **LESS THE NOISE MORE THE OUT SIGNAL.**
-
- **NOISE LOSS OF INFORMATION.**
- **LOSS OF INFORMATION UNCERTAINTY.**

QUALITY OF DATA

- UNCERTAINTY DEPENDS ON THE QUALITY OF DATA.
- ASSESSMENT OF QUALITY.
- COVARIANCE MATRIX.
- GENERATION OF COVARIANCE MATRIX USING QUALITY DATA.

BASIC STATISTICS

- MEAN = μ = < x >
- dx = x - μ_x, dy = y- μ_Y, then
- VARIANCE = < dx dx >
- STD.DEV = SQRT [VARIANCE]
- COVARIANCE = < dx dy >
- ρ = < dx dy >⁄SD[x] SD[y]

COUNTING EXPERIMENT

TWO SOURCES 1 AND 2.

N1 = G1 – B = 900 – 700 = 200
N2 = G2 – B = 981 – 700 = 281

SD(N1) = SQRT(900 + 700) = 40.
SD (N2) = SQRT(981 + 700)= 41.

RSD(N1) = SD(N1)/N1 = 0.2
RSD(N2) = SD(N2)/N2 = 0.146.

ERROR (VARIANCE) MATRIX

- Var (G1, G2, B) =

900		
	981	
		700

COV. AND COR. MATRIX

- COV(N1, N2)
- COR(N1, N2)

1600 1.0	700 0.427
700 0.427	1681 1.0

TYPES OF ERROR

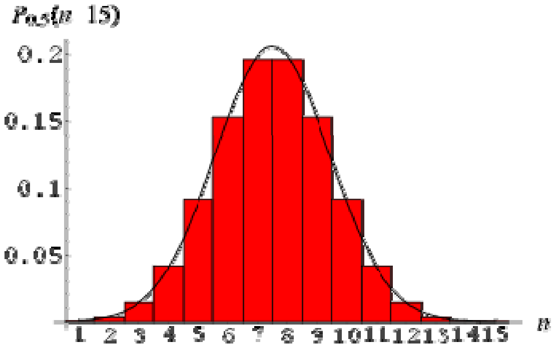
- RANDOM ERROR:
- STOCHASTIC FLUCTUATIONS.
- HAS A SECOND MOMENT.
- UNCORRELATED.

- SYSTEMATIC ERROR:
- REMAINS CONSTANT.
- NO SECOND MOMENT.
- CORRELATED.

RANDOM ERROR

- REDUCED BY REPETITION.
- STATISTICAL TECHNIQUES.
- NO LIMITS FOR THE RANDOM ERROR.

NORMAL DISTRIBUTION



SYSTEMATIC ERROR

- CANNOT BE REDUCED BY REPETITION.
- NEED FOR LIMITS FOR SYSTEMATIC ERROR.
- MATHEMATICAL MODEL.

REQUIREMENTS OF CENTRAL LIMIT THEOREM

- VARIABLES SUMMED MUST BE INDEPENDENT.
- ALL VARIABLES MUST HAVE FINITE MEAN AND VARIANCE.
- NO VARIABLE CAN MAKE AN EXCESSIVELY LARGE CONTRIBUTION TO THE SUM.

SYSTEMATIC ERROR

- ENTROPY BASED APPROACH.

- $H = \int f(x) \log [f(x)]\, dx$.
- LARGER THE ENTROPY, GREATER THE UNCERTAINTY.
- MATHEMATICAL MODEL.
- DETERMINANT INEQUALITIES.

SOURCES OF DISCREPANCY IN REACTOR CALCULATIONS

- MODELLING.
- CALCULATIONAL METHODS.
- MONTE CARLO.
- TRANSPORT THEORY

- NEUTRON X-SECTIONS.
- FOCUS ON X-SECTIONS.

NEUTRON X-SECTIONS

- **<u>ABSOLUTE</u>**
- **REACTION RATE = C = $N\sigma\varphi$;**
- **$\sigma = \{ C / N \varphi \}$**
- **<u>RELATIVE</u>.**
- **$[\sigma_i / \sigma_j] = [C_i / C_j] [N_j / N_i]$**

RATIO OF COUNT RATES

- **SQRT {[RSD(N1)]^2 + [RSD(N2)]^2}= 0.25**
- **SQRT{[RSD(N1)]^2 + [RSD(N2)]^2– CF]} =0.19**
- **CF = 2.ρ.RSD[N1] RSD[N2]**

RELATIVE X-SEC. MEASUREMENT

- **$[\sigma_i / \sigma_j] = [C_i / C_j] [N_j / N_i]$**
- **$\sigma = \sigma (p_i, p_j , p_k, \ )$**
- **M_σ REL. COV.MATRIX OF σ**
- **M_p REL. COV.MATRIX OF p**
- **$M_\sigma = B\, M_p\, B^T$ LAW OF ERROR PROPAGATION.**
- **INF. M_σ DEPENDS ON INF. M_p**

MODEL TO REDUCE SYSTEMATIC UNCERTAINTY.

- **UNCTY. M_σ DEPENDS ON UNCTY M_p**
- **MATHEMATICAL MODEL.**
- **DETERMINANT INEQUALITIES.**
- **REDUCE UNCTY. M_p**

PARAMETERS IN X-SEC MEASUREMENT

p_1	**C**	**Count Rates,**
p_2	**ε**	**Efficiency of the detector,**
p_3		**Geometrical Area,**
p_4		**Half life,**
p_5		**Back scattering,**

DETERMINANT INEQUALITIES

- **<u>SIGN OF THE DET</u>.**
- **Det.M_p> 0 FOR ρ = 0.**
- **Det.M_p =0 FOR ρ ± 1.**
- **Det.$M_p \geq 0$**

σ_X^2	ρ_{XY} $\sigma_X \sigma_Y$	ρ_{XZ} $\sigma_X \sigma_Z$
	σ_Y^2	ρ_{YZ} $\sigma_Y \sigma_Z$
	ρ_{ZY} $\sigma_Z \sigma_Y$	σ_Z^2

ESTIMATION OF BOUNDS

- LIMITS FOR SYSTEMATIC ERROR.
- **Let q Constant Bias.**
- **Det. ρ (q) ≥ 0.**

1.0	ρ_{12}(q)	ρ_{13}(q)
	1.0	ρ_{23}(q)
	ρ_{32}(q)	1.0

- **ALGORITHM.**
- **PROCESS HIGHER ORDER MATRICES.**

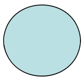
SIMPLE EXAMPLE

- **Det.M_P = [1– ρ^2]**
- **SMALLER THE ρ, HIGHER THE Det.M_P.**
- **ρ IS ESTIMATED BY BOUNDS.**

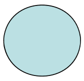
1.0	**ρ**
ρ	**1.0**

INFORMATION THEORY

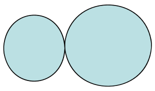
- **H(X)**



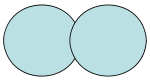
- **H(Y)**



- **H(X,Y)**



INFORMATION THEORY



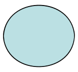
- **MUTUAL INFORMATION MIF(X,Y)**
- **H(X,Y) =**
- **H(X)+H(Y) -MIF(X,Y)**

MUTUAL INFORMATION

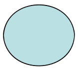
- **H(X,Y) =**
- **H(X)+H(Y) -MIF(X,Y)**
- **IF MIF(X,Y) = 0,**
- **H(X,Y) = H(X) + H (Y)**
- **IF MIF(X,Y) > 0,**
- **H(X,Y) < H(X) + H(Y)**

UNCERTAINTY REDUCTION AND MIF

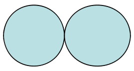
- **H(RANDOM)** =



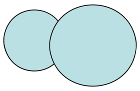
- **H(SYSTEMATIC)** =



- **H(TOTAL)**
- **= H(RAN) + H(SYS)**
- **WHEN MIF =0**



UNCERTAINTY REDUCTION AND MIF



- **WHEN MIF > 0,**
- **H(TOTAL)**

< H(RAN) + H (SYS)

MIF AND DET.M_p

- **MIF(X,Y) = CONST. DET.M_p.**
- **HIGHER THE VALUE OF DET.M_p, HIGHER THE MIF(X,Y)**
- **VALUE OF DET. M_p IS MADE HIGHER BY USING**
- **EITHER UB OR LB FOR CORRELATED ELEMENTS.**

APPLICATION I

- **REDUCE OPTICAL MODEL DEFICIENCY.**
- **NUCLEAR MODEL CALCULATIONS.**
- **UTILITY OF OPTICAL MODEL.**

- **OPTICAL MODEL PARAMETERS (OMP).**
- **U + iW, $R=R_0A^{1/3}$, a.**

- **OMP ARE HIGHLY CORRELATED.**
- **^{239}PU IN JENDL 3.2.**

^{239}PU OMP CORR.MATRIX

U 0.95	**1.0**			
R 0.99	**-0.966**	**1.0**		
W 4.23	**-0.558**	**0.492**	**1.0**	
a 4.92	**-0.153**	**0.292**	**-0.294**	**1.0**

VALUES OF BOUNDS AND THEIR DETERMINANT FOR OMP

EX.VAL.	L.Bound	U.Bound	Det.LB	Det.UB
-0.966	-0.989	0.137		
-0.558	-0.7	-0.373	-0.574	**0.186** (0.024)
-0.153	-0.429	-0.038		

APPLICATION I.

- **SIMULATION OF OPTICAL MODEL PARAMETERS WITH REDUCED MODEL DEFICIENCY BY D-OPTIMAL CRITERION.**
- **P.T.KRISHNA KUMAR AND HIROSHI SEKIMOTO- ACCEPTED IN ANNALS OF NUCLEAR ENERGY (2009).**

APPLICATION II

- **GENERATION OF ROBUST RESONANCE PARAMETERS (RP).**
- **AVERAGED CROSS-SECTION.**
- **HIGHLY CORRELATED.**
- **PITFALLS IN SAMMY (ORNL) AND IN**
  KALMAN (KYUSHU UNIVERSITY).

KALMAN FOR JENDL 3.2

- **$P = X - XC^T \{CXC^T + V\}^{-1} CX$**

- **P= FINAL COV.MATRIX OF RP.**
- **X= INITIAL COV.MATRIX OF RP.**
- **C= SEN.MATRIX.**
- **V=COV.MATRIX OF AVG.X-SEC.**

CORR.MATRIX FOR ^{235}U AVERAGED X-SEC.

Energy Range (KeV)	Uncert. (%)	Exis.Val	U.Bound	L.Bound
600-700	1.26	0.941	0.999	0.820
700-800	1.26	0.937	0.999	0.815
800-900	1.28	0.924	0.997	0.798

APPLICATION II

- **AN INFORMATION THEORY APPROACH TO MINIMIZE CORRELATED SYSTEMATIC UNCERTAINTY IN MODELLING RESONANCE PARAMETERS.**
- **P.T.KRISHNA KUMAR AND HIROSHI SEKIMOTO, APPLIED RADITION AND ISOTOPES, Vol:67, 329-333, (2009).**

APPLICATION III

- **MAXIMIZATION OF REPRESENTATIVITY FACTORS (RF)**
- **COMPARISON OF NUCLEAR SYSTEMS.**
- **$\delta R^2_{new} = \delta R^2_{old} \{1 - RF^2\}$**

- **WHEN RF =0, THEN, $\delta R^2_{new} = \delta R^2_{old}$**
- **WHEN RF > 0, THEN $\delta R^2_{new} < \delta R^2_{old}$**

CORR.MATRIX MINOR ACTINIDES

^{241}Am	1.0	0.94	0.94	0.62
^{242}Am		1.0	0.99	0.61
^{242}Cm			1.0	0.61
^{244}Cm				1.0

BOUNDS FOR MINOR ACTINIDES

EXI.VAL.	UB	LB	Det.UB	DET.LB
0.94	0.98	0.88		
0.94	0.98	0.88	9.3E(-6)	**0.024** (0.001)
0.62	0.84	0.31		

UNCERTAINTIES IN X-SECTION MEASUREMENT

σ	**1**	**2**	**3**	**Corr.Cff**
C	**0.5**	**1.0**	**0.3**	
ε	**1.6**	**2.2**	**1.3**	**ρ_{12} = 0.8** **ρ_{13} =0.5** **ρ_{23} =0.9**
B	**2.0**	**2.0**	**2.0**	**ρ = 1.0**

COV. AND COR. MATRIX

- **COV.M M_σ**
- **COR.M ρ**

- **RATIO OF X-SEC.**
- **WITHOUT COVARIACE =4.20%**
- **WITH**
- **COVARIANCE=2%**

7.81	**6.82** **0.77**	**5.04** **0.80**
	9.84	**6.57** **0.87**
		5.78

CORRELATED ELEMENTS

ELEMENT	EXISTING VALUE	LOWER BOUND VALUE
ρ_{12}	0.77	0.32
ρ_{13}	0.80	0.37
ρ_{23}	0.87	0.16

APPLICATION III

- **MAXIMIZATION OF REPRESENTATIVITY FACTORS FOR EXPERIMENTAL PLANNING OF CROSS-SECTION MEASUREMENTS.**
- **P.T.KRISHNA KUMAR AND HIROSHI SEKIMOTO, ANNALS OF NUCLEAR ENERGY, VOL:35, 2243-2248, (2008).**

APPLICATION IV

- **TRANSMISSION MEASUREMENT OF IRON.**
- **$I = I_0\, e^{-N\sigma x}$.**
- **INTIAL INTENSITY I_0**
- **FINAL INTENSITY AFTER TRANSMISSION I**
- **THICKNESS x.**

- **$\sigma = [\,1/Nx]$ LOG.$[\,I_0/I\,]$**
- **DEPENDS ON REDUCTION PARAMETERS.**
- **NEUTRON COUNTS WITH AND WITHOUT SAMPLE, BACKGROUND, DEAD TIME, etc.**

APPLICATION IV

ENERGY (MeV)	CROSS-SECTION. (Barns)	ORIGINAL SYS.UNCY (%)	SYS.UNCY BY MIF. (%)
2.4-3.0	3.424	0.018	0.011
3.0-4.5	3.531	0.018	0.014
4.5-8.0	3.586	0.017	0.014

APPLICATION IV

- REDUCTION OF SYSTEMATIC UNCERTAINTY IN TRANSMISSION MEASUREMENT OF IRON BY ENTROPY BASED MUTUAL INFORMATION.
- P.T.KRISHNA KUMAR AND HIROSHI SEKIMOTO, RADIATION MEASUREMENTS, 2009 (IN PRINT).

APPLICATION V

- REDUCTION OF SYSTEMATIC UNCERTAINTY IN RADIOPHARMACEUTICAL ACTIVITY.

- NEUTRON GENERATORS.
- ACTIVATION ANALYSIS.
- ^{99m}Tc , ^{113m}In.
- UNCERTAINTY IN FORMATION.

APPLICATION V

- ^{252}Cf.
- ^{113}In(n,n\`)^{113m}In.

- $C = N\sigma\varphi$ ∏ SYS.UNCY.
- SYS.UNCY INTRODUCE NOISE.

- MINIMIZE THE NOISE BY REDUCTION OF SYSTEMATIC UNCERTAINTY.

APPLICATION V

- REDUCTION OF SYSTEMATIC UNCERTAINTY IN RADIOPHARMACEUTICAL ACTIVITY BY ENTROPY BASED MUTUAL INFORMATION.
- P.T.KRISHNA KUMAR AND HIROSHI SEKIMOTO, ITSURO KIMURA.
- TO APPEAR IN NUCLEAR INSTRUMENTS AND METHODS IN PHYSICS RESEARCH.

APPLICATION VI

- DECIPHERING ROBUST REACTOR KINETIC DATA.
- MEASUREMENT OF KINETIC PARAMETERS FOR AGCR.
- 665 Mwe, UO_2, 2.5% ENRICHED,
- CO_2, GRAPHITE MODERATOR.
- PITFALLS IN CHAUVENT’S CRITERION.
- REJECT DATA WITH HIGH CORRELATION COEFFICIENT.

CORRELATION MATRIX

COEFF.	FTC	HTFC	HTMC
FTC	1.0	0.849	0.373
HTFC		1.0	0.754
HTMC			1.0

CORRELATION COEFFICIENTS

EXISTING VALUE	LOWER BOUND VALUES	DET.M_p
0.849	-0.328	
0.373	0.293	0.8097
-0.754	-0.174	0.0491

APPLICATION VI

- DECIPHERING ROBUST REACTOR KINETIC DATA USING MUTUAL INFORMATION.
- P.T.KRISHNA KUMAR, ANNALS OF NUCLEAR ENERGY, Vol:34, 201-206, 2007.

APPLICATION VII

- CLASSIFICATION OF RADIO ACTIVE ORES.
- AERIAL SURVEY.
- MOBILE COUNTING USING NaI(TI) DETECTORS.
- SIMILARITY MEASURE.
- HIGHER CORRELATION COEFFICIENT.

APPLICATION VII

- CLASSIFICATION OF RADIO ELEMENTS USING MUTUAL INFORMATION: A TOOL FOR GEOLOGICAL MAPPING.
- P.T.KRISHNA KUMAR, V.PHOHA AND S.S.IYENGAR, INTERNATIONAL JOURNAL OF APPLIED EARTH OBSERVATION AND GEOINFORMATION, Vol:10, 305-311, (2008).

APPLICATION VIII

- DESIGN OF SENSORS.
- DESIGN OF DISCRIMINATING TASTE SENSORS USING MUTUAL INFORMATION.
- DESIGN OF DRUGS.
- SALT(NaCl), SOUR(HCL), BITTER (QUININE),SWEET(SUCROSE),UMAMI(MSG)
- P.T.KRISHNA KUMAR, SENSORS AND ACTUATORS (CHEMICAL B), B 119, 215-219, (2006).

ADVANTAGES OF MIF

- **IMPROVE EXISTING VALUES.**
- **IMPROVE SYSTEMATIC UNCERTAINTY (CORRELATED).**
- **NO ASSUMPTIONS ABOUT DISTRIBUTION OF THE DATA.**
- **DISCRIMINATE STATISTICAL AND SYSTEMATICAL**.

ADVANTAGES OF MIF

- **ELEMENT WISE PROCESSING.**
- **ENTIRE STRUCTURE OF THE COVARIANCE MATRIX.**
- **AID EXPERIMENTALISTS TO IMPROVE METHOD OF MEASUREMENT AND INSTRUMENTATION.**
- **ROBUST AND FAST PROCEDURE.**

- THANK YOU.